ROLPHTON

NUCLEAR TRAINING CENTRE

COURSE 221

NUCLEAR TRAINING COURSE

COURSE 221

2 - Level
2 - Science Fundamentals
1 - MATHEMATICS

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Integral Calculus

Appendices

Mathematics - Course 221

OBJECTIVES

221.10-1 Basic Reliability Concepts

1. Given P(A) and P(B), the probabilities of independent events A and B, respectively, calculate P(A and B) and P(A or B), using the formulas:

   $$P(A \text{ and } B) = P(A)P(B), \text{ and}$$

   $$P(A \text{ or } B) = P(A) + P(B) - P(A)P(B).$$

2. Define (a) independent events
   (b) reliability
   (c) unreliability
   (d) unavailability of a safety system.

3. Given reliability R, calculate unreliability, Q, and vice versa.

4. State two methods of improving reliability of safety systems.

5. Calculate component failure rate, λ, given a total number of failures amongst a given number of components during a given time interval.

6. Calculate the test interval, T, in years, given the test frequency in tests per shift, day, week, month, or year.

7. Given information determining any two of the variables Q, λ, T, calculate the third variable for a tested safety system.

8. Given information determining the failure rate of the regulating system and the unavailability of the protective system, calculate the annual risk of a reactor power excursion.

9. Apply the above principles to calculate the unreliability of a network of components, given information determining the unreliabilities of the network components.

221.20-1 The Straight Line

1. Define:
   (a) slope of a line
   (b) rise of a line segment
   (c) run of a line segment
   (d) angle of inclination of a line

2. Write down the relationship between

   (a) slope m, rise, and run
   (b) slope m, and angle of inclination, θ

3. State the significance to orientation of a line if the line slope is

   (a) positive
   (b) negative
   (c) zero
   (d) undefined

4. Calculate the slope of a line, given

   (a) two points on the line
   (b) the slope of a parallel line
   (c) the slope of a perpendicular line
   (d) the equation of the line
   (e) the rise and the run of a segment of the line
   (f) the angle of inclination of the line

5. Given the slope of a line, calculate the change in y corresponding to a given change in x, and vice versa.

6. Identify whether the equation of a line is given in general or slope-intercept form, and convert from the one form to the other.

7. Find the equation of a line, given

   (a) two points on the line
   (b) one point on the line and the slope
   (c) the slope of the line and the y-intercept

8. Graph a line given its equation.

221.20-2 The Derivative

1. State that for a linear function f(x) the following are equivalent:

   (a) the slope
   (b) the 'instantaneous' rate of change of f with respect to x at any point on the graph, y = f(x).
   (c) the average rate of change of f with respect to x over any x-interval.

2. Define the derivative of a function f(x).

3. Recognize and use the notation:

   (a) $\frac{dy}{dx}$ (b) f'(x)

4. State that the graphical significance of f'(x) is that f'(x) is the slope of the tangent to the curve y = f(x) at (x,f(x)).

5. State and apply the rules for differentiating the following:

   (a) x^n
   (b) cf(x)
   (c) c
   (d) f(x) ± g(x)

221.20-3 Simple Applications of Derivatives

1. Given the function f(x), find

   (a) the slope, and
   (b) the equation of the tangent and normal to the curve y = f(x) at any given point (x_1, y_1) on the curve.

2. Differentiate a given polynominal displacement function to obtain the corresponding velocity function.

3. Differentiate a given polynominal velocity function to obtain the corresponding acceleration function.

221.20-4 Differentiating Exponential Functions

1. Differentiate functions of the form

   (a) $f(x) = ke^{g(x)}$
   (b) $f(x) = P(x) \pm ke^{g(x)}$

   where k is a constant, and g(x) and P(x) are both polynominals.

2. Given the nuclear decay formula, $N(t) = N_0e^{-\lambda t}$, prove that

   (a) $\frac{dN}{dt} = -\lambda N$

   (b) $A(t) = A_0e^{-\lambda t}$, where $A = -\frac{dN}{dt}$

3. Given any two of the variables A, λ, N (activity, decay constant, number of radioactive nuclei, respectively), calculate the third variable.

4. Given any three of the following variables, calculate the fourth variable:

   (a) N, N_0, λ, t (see nuclear decay formula above)
   (b) A, A_0, λ, t (see activity decay formula above)
   (c) P, P_0, T, t (see power growth formula below)

5. Given the reactor power growth formula $P(t) = P_0e^{t/T}$ prove that

   (a) $\frac{dP}{dt} = \frac{P}{T}$

   (b) $\frac{d}{dt} \ln P = \frac{1}{T}$

6. State the advantage of

   (a) a log power signal (over a linear power signal) for power indication and control
   (b) a rate log power signal for reactor protection.

221.20-5 The Derivative in Science and Technology

1. Translate a given verbal rate-of-change statement into a differential equation, and vice versa.

2. Given a sketch showing the fluctuation of a controlled parameter about set point, sketch on the same time axis, typical corresponding proportional component, derivative component, and total response of a proportional-derivative controller.

3. For the case of tank level control via regulation of inflow, sketch typical level fluctuations following a step change in outflow for

   (a) proportional only control
   (b) proportional plus derivative control

4. State two advantages of adding a derivative component to proportional control.

221.30-1 The Integral

1. State that integration is the opposite of differentiation.

2. Recognize and use the integral notation.

3. Integrate functions of the following forms:

   (a) $f(x) = 0$
   (b) $f(x) = x^n$
   (c) $f(x) = e^{f(x)} f'(x)$
   (d) $f(x) = g(x) \pm h(x)$

4. Given an acceleration function, obtain the corresponding velocity and displacement functions by integration.

5. Given a velocity function, integrate to obtain the corresponding displacement function.

6. Given the equation of a curve, $y = f(x)$, find the area under the curve in the interval $x = a$ to $x = b$ by evaluating the appropriate definite integral.

221.30-2 Applications of The Integral as an Infinite Sum

1. Find the area between two curves (one of which could be an axis) by applying the 'slice technique', including a diagram showing representative slice.

2. Given force F as a function of displacement x, calculate the work done by this force acting through x = a to x = b.

3. Given a sketch showing the fluctuation of a controlled parameter about set point, sketch on the same time axis typical corresponding proportional component, reset component, and total response of a proportional-integral controller.

4. For the case of tank level control via regulation of inflow, sketch typical level fluctuations following a step change in outflow for

   (a) proportional only control
   (b) proportional plus reset control.

5. State the advantage of adding a reset component to proportional control.

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