5. Approximate Solutions for Reactivity Ramps

- **Reactivity Mechanisms**
  - characterized by:
    1. **reactivity worth.** (mk) \( \Delta \rho = \lambda_o - \lambda_1 \)
      
      Must be greater than the reactivity effect to be compensated.
    
    2. **insertion rate** (mk/s) \( \Delta \rho / \Delta t \)
      
      Must be as rapid as the effect to be controlled.
    
    - The reactivity worth is obtained from the difference of eigenvalues between 2 diffusion calculations:
      
      \[
      \delta \rho_{rod} = \rho_1 - \rho_0 \\
      = (1 - \lambda_1) - (1 - \lambda_o) \\
      = \lambda_o - \lambda_1 \\
      = -\Delta \lambda
      \]
    
    - It is extremely difficult to obtain an analytical solution to the P.K. equations when reactivity varies with time. A numerical solution is usually obtained. For an analytical solution, an approximation is required.
    
    - We will consider a Reactivity Ramp: \( \rho(t) = \mu \cdot t \)
### CANDU Reactor Control

**Inherent Reactivity Effects in CANDU 600**

<table>
<thead>
<tr>
<th>CAUSE</th>
<th>REACTIVITY WORTH</th>
<th>INSERTION RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel and coolant warm-up:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(25 °C to 290 °C)</td>
<td>fresh fuel: - 7 mk</td>
<td>seconds, minutes</td>
</tr>
<tr>
<td></td>
<td>equilibrium: +3 mk</td>
<td></td>
</tr>
<tr>
<td>Reactor power: 0 to 100% FP</td>
<td>fresh fuel: - 9.4 mk</td>
<td>seconds, minutes</td>
</tr>
<tr>
<td></td>
<td>equil.: - 4.5 mk</td>
<td></td>
</tr>
<tr>
<td>Moderator temperature: 40 °C to 70 °C</td>
<td>fresh fuel: - 2.1 mk</td>
<td>hours (seasons)</td>
</tr>
<tr>
<td></td>
<td>equil.: + 2.7 mk</td>
<td></td>
</tr>
<tr>
<td>Fuel burnup:</td>
<td>≈ 20 mk</td>
<td>2-3 months (continuous)</td>
</tr>
<tr>
<td>- initial excess reactivity</td>
<td>- 0.4 mk/day</td>
<td>hour</td>
</tr>
<tr>
<td>- reactivity decline</td>
<td>0.35 mk</td>
<td></td>
</tr>
<tr>
<td>- refueling one channel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coolant density: 0.8 → 0 g/cc</td>
<td>fresh fuel: + 15 mk</td>
<td>0.1 - 2 s</td>
</tr>
<tr>
<td>(LOCA)</td>
<td>equil.: + 10 mk</td>
<td></td>
</tr>
</tbody>
</table>

**Reactivity Mechanisms in CANDU 600**

<table>
<thead>
<tr>
<th>TYPE</th>
<th>REACTIVITY WORTH</th>
<th>INSERTION RATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor Regulating System</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- primary: 14 Liquid Zone Controllers</td>
<td>6 mk</td>
<td>±0.14 mk/s</td>
</tr>
<tr>
<td>- secondary: 4 Solid Control Rods</td>
<td>10 mk</td>
<td>±0.09 mk/s</td>
</tr>
<tr>
<td>Long Term Reactivity Control</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 21 Adjuster Rods (7 banks)</td>
<td>15 mk large</td>
<td>±0.2 mk/s hours, days</td>
</tr>
<tr>
<td>- Soluble Poison (boron + gadolinium)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emergency Shutdown Systems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 28 Shut Off Rods</td>
<td>-60 mk</td>
<td>1.3 s</td>
</tr>
<tr>
<td>- Gadolinium Injection</td>
<td>very large</td>
<td>2.5 s</td>
</tr>
</tbody>
</table>
Approximate Solutions

Constant Delayed Source (CDS) Approximation

- Delayed source does not vary rapidly: we assume it does NOT vary at all over a limited period of time \((t \text{ is small})\)

\[
\begin{align*}
\sigma_{d}^{\text{CDS}}(t) &= s_{d0} \\
&= \sum_{k=1}^{K} \lambda_{k} \frac{\beta_{k}}{\lambda_{k}} p_{o} \\
&= \beta p_{o}
\end{align*}
\]

- We obtain:

\[
\Lambda \frac{dp}{dt} = [\rho(t) - \beta] p(t) + s_{do} + s(t)
\]

- Considerably more simple than original equations, even with only 1 delayed neutron group (1DG).

- The CDS equation can be integrated for an arbitrary variation of \(\rho(t)\):

\[
\rho(t) = e^{\int_{0}^{t} \left( \frac{\rho(\tau) - \beta}{\Lambda} \right) d\tau} \left\{ p_{o} + \int_{0}^{t} \frac{s_{do} + s(t')} \Lambda \right\} \left[ e^{-\int_{0}^{t} \frac{\rho(\tau) - \beta}{\Lambda} d\tau} \right] dt'
\]

- When reactivity is constant (step insertion at \(t = 0\))

\[
\rho(t) = e^{\left( \frac{\rho - \beta}{\Lambda} \right) t} \left\{ p_{o} + \int_{0}^{t} \frac{s_{do} + s(t')} \Lambda \right\} \left[ e^{-\left( \frac{\rho - \beta}{\Lambda} \right) t} \right] dt'
\]
Prompt Jump Approximation (PJA)

- Response to step-change in reactivity showed one component responding very rapidly, while all other components were related to the delayed neutron source.

- Assume the prompt response to be essentially instantaneous:

\[
T_p = \frac{\Lambda}{\beta - \rho} \rightarrow 0 \quad \text{with PJA}
\]

\[
\Lambda \rightarrow 0
\]

- PJA response is always ahead of the real solution:
Prompt Jump Approximation (cont'd)

- The amplitude equation can be written:

\[
\frac{dp}{dt} = \frac{\rho(t) - \beta}{\Lambda} p(t) + \frac{s_d(t)}{\Lambda} + \frac{s(t)}{\Lambda}
\]

- Making \( \Lambda \to 0 \):

\[
p_{\rho j}(t) = \frac{s_d(t) + s(t)}{\beta - \rho(t)}
\]

(see source multiplication formula)

- One Delayed Group:

\[
s_d(t) = \lambda \zeta(t)
\]

\[
\frac{d\zeta}{dt} = \beta p(t) - \lambda \zeta(t)
\]

1. from above:

\[
[p(t) - \beta] p(t) + \lambda \zeta(t) + s(t) = 0
\]

2. derive with \( t \):

\[
[p(t) - \beta] \frac{dp}{dt} + \frac{d\rho}{dt} p(t) + \lambda \frac{d\zeta}{dt} + \frac{ds}{dt} = 0
\]

3. eliminate \( d\zeta/dt \):

\[
\frac{dp}{dt} = \left[ \frac{\lambda \rho(t) + d\rho/dt}{\beta - \rho(t)} \right] p(t) + \left[ \lambda s(t) + \frac{ds}{dt} \right]
\]

4. initial condition:

\[
p^0 = p(0^+) = \left( \frac{\beta - \rho_0}{\beta - \rho^+} \right) p_0
\]
• general solution (PJA):

\[
\rho_p(t) = \exp \left[ \int_0^t \left( \frac{\lambda \rho(t') + dp/dt'}{\beta - \rho(t')} \right) dt' \right].
\]

\[
\left\{ p^o + \int_0^t \left( \frac{\lambda s(t') + ds/dt'}{\beta - \rho(t')} \right) \exp \left[ -\int_0^t \left( \frac{\lambda \rho(t) + dp/dt}{\beta - \rho(t)} \right) dt' \right] \right\}
\]

Log-Rate

• The Log-Rate (or logarithmic derivative) can be measured.

• For an initially critical reactor (no source s):

\[
\tau(t) = \frac{1}{p} \frac{dp}{dt} = \frac{\lambda \rho(t) + dp/dt}{\beta - \rho(t)}
\]

• important observations:

1. rate of change of power in a nuclear reactor is not only function of the dynamic reactivity \( \rho(t) \). It is also a function of the rate of change of reactivity \( dp/dt \). \( \Rightarrow \) denotes the importance insertion rate (speed) of reactivity mechanisms.

2. power can decrease even if reactor is supercritical \( (\rho(t) > 0) \), if:

\[
\frac{dp}{dt} < -\lambda \rho
\]

3. power can increase even if reactor is subcritical \( (\rho(t) < 0) \), if:

\[
\frac{dp}{dt} > -\lambda \rho
\]

• Concl.:

When reactivity varies continuously, reactor response is sensitive to the rate of change of reactivity.
Application of PJA to a Reactivity Ramp

\[
\rho(t) = \begin{cases} 
0 & t \leq 0 \\
\rho_0 + \mu t & t > 0^+ 
\end{cases}
\]

- The solution can be expressed as:

\[
\rho_{pj}(t) = \rho_0 e^{l_1(t)} \cdot e^{l_2(t)}
\]

where:

\[
l_1(t) = \int_0^t \frac{d\rho}{t + \beta - \rho(t)} \, dt
\]

\[
= \int_{\rho_0}^{\rho(t)} \frac{d\rho}{\beta - \rho}
\]

\[
= -\ln \left( \frac{\beta - \rho(t)}{\beta - \rho_0} \right)
\]

\[
l_2(t) = \int_0^t \frac{\lambda \rho(t')}{t + \beta - \rho(t')} \, dt'
\]

\[
= \int_0^t \frac{\lambda \rho_0}{\beta - \rho_0 - \mu t'} \, dt' + \int_0^t \frac{\lambda \mu t'}{\beta - \rho_0 - \mu t'} \, dt'
\]

\[
= \frac{\lambda \rho_0 \ln \left( \frac{\beta - \rho_0}{\beta - \rho(t)} \right)}{\mu} + \lambda \left[ \frac{\beta - \rho_0 \ln \left( \frac{\beta - \rho_0}{\beta - \rho(t)} \right)}{\mu} - t \right]
\]

\[
= \frac{\lambda \beta}{\mu} \ln \left( \frac{\beta - \rho_0}{\beta - \rho(t)} \right) - \lambda t
\]
Application of PJA to a Reactivity Ramp (cont'd)

• we find:

\[
p_{pj}(t) = p_o \left[ \frac{\beta}{\beta - \rho(t)} \right] e^{-\lambda t} \left( \frac{\beta - \rho_o}{\beta - \rho(t)} \right)^{\frac{\lambda \beta}{\mu}}
\]

• in the absence of an initial step \( \rho_o \):

\[
p_{pj}(t) = p_o e^{-\lambda t} \left( \frac{\beta}{\beta - \mu t} \right)^{t+\frac{\lambda \beta}{\mu}}
\]

• if \( \mu \) is zero (no ramp), with an initial step \( \rho_o \), we find:

\[
p_{pj}(t) = p_o \left( \frac{\beta}{\beta - \rho_o} \right)^{\frac{\lambda \rho_o}{\mu} t}
\]

• Conclusion:

For constant reactivity (no ramp), the PJA reduces to the first term of the equation obtained with 1DG.
Reactivity Ramp in a CANDU Reactor
(0.5 mk/s)

- with an initial step of 3 mk:
Small Reactivity Ramps in a CANDU Reactor

\[ \mu = 0.05 \text{ mk/s} \]

\[ \mu = 0.005 \text{ mk/s} \]
Large Reactivity Ramp in a CANDU Reactor
(LOCA, 3.5 mk/s)

- **Observations:**

1. PJA is not applicable (solution diverges as $\rho \Rightarrow \beta$)

2. CDS gives good results as long as $\rho < \beta$

3. For fast ramps, number of delayed groups is not significant (although the presence of the delayed source is important)
Assignment no 6

From the definition of the Point Kinetics Parameters, show that, for the extremely simplified case where:
1. the nuclear properties are uniform;
2. the reactor contains only one fissile isotope;
3. the neutrons are monoenergetic.

a) the effective delayed neutron fraction becomes:  \[ \beta = \frac{\nu \lambda}{v} \]
b) the mean generation time can be written:  \[ \Lambda = \frac{1}{\nu \Sigma f} = \Delta t / \nu \]
where \( \Delta t \) is the mean time between fission events.

Assignment no 7

A CANDU reactor under equilibrium refueling is operating at full power when the Safety Shutdown System is activated. A negative reactivity of -30 mk is suddenly inserted, and the power drops very rapidly. The neutron power, as indicated by the control flux detectors, does not fall to zero;

a) the power rapidly drops to a particular value, and afterwards, goes down at a much slower pace. Why does the power stop falling rapidly? At what power level (in % of full power, %FP) does the power tend to stabilize?

b) what is the neutron power in the reactor after 1 minute?

Assignment no 8

The previous reactor has been shutdown for some time. The ionization chambers indicate a neutron power of \( 1.5 \times 10^{-3} \) %FP. The operator demand a 50% power increase to the Reactor Regulating System (RRS). After a few minutes, power is stable at the new level, and the operators notices that the Liquid Zone Controler level has dropped by 15% and is now indicating 55% fill.

What will be the final Liquid Zone Control level when the reactor becomes critical?

(Assume that 100% displacement of LZC is worth 6 mk in reactivity)