Dynamics of the Subcritical Core

Neutron Sources
Subcritical Multiplication of Sources
Response Time to $+\Delta k$

What do Sources Do?

- There is measurable flux in a shut down reactor core, even if the reactor in the Guaranteed Shutdown State (GSS), with very large amounts of neutron absorbing stuff in the core.
- This is caused by subcritical multiplication of source neutrons.
Why is there Flux in a Shut Down Reactor? How is it possible to get the first fission?

- A REACTOR, ONCE STARTED, CANNOT BE SHUT DOWN
- We refer to the reactor as:
  - in the operating state
  - in the shut down state
- But the Shut Down Reactor Core is not de-energized like other machines:
  - there is heat from fission product decay
  - there is flux from subcritical source multiplication

What is a Neutron Source?

- A neutron source injects a steady supply of neutrons, independent of power level, temperature, or any controllable parameter.
- CANDU reactors have two important natural sources:
  - spontaneous fission of U-238;
  - photo-neutrons released from deuterium nuclei in the heavy water by energetic $\gamma$-rays from fission product decay.
Spontaneous Fission

We will look at this source first in some detail.
The size of this source is about $10^{-14}$ of full power flux; ♠ $1 \text{n}\,\text{cm}^{-2}\,\text{s}^{-1}$.

Spontaneous Fission of U-238

- Each CANDU fuel bundle has, initially, about 18.9 kg of natural uranium, 99.3% U-238.
- A CANDU-6 has 380 fuel channels & 12 bundles in each channel, over 85 Mg of U-238.
- This large quantity overwhelms any spontaneous fissions of other isotopes in the fuel.
  - Compare the U-235 content of fresh fuel (0.7%)
  - This is just over 600 kg of fissile fuel in a fresh core.
Spontaneous Fission Source Strength

◆ S.F. decay of U-238 is a very rare process.
◆ The Half-Life is $T_{1/2} = 8 \times 10^{15}$ years
◆ This results in over 1/2 million decays per second in the whole core, but,
◆ The neutrons are quickly absorbed, in about a milli-second, so
◆ At any instant in the core there are just over 1000 neutrons from spontaneous fission.

Neutron Density & Neutron Flux

◆ The Volume of the CANDU core, a cylinder about 6 m long, diameter 7 m, is over 200 m$^3$
◆ 1000 neutrons is 5 neutrons per cubic meter!
  – Most neutrons slow down before they are absorbed
◆ Thermal neutron speed is about 2200 m/s
◆ Flux ($\phi$) is (neutron density $\times$ speed)
  $\phi = 5 \times 2200 \times 10^{-4} \text{ cm}^{-2} \text{ s}^{-1}$
This is approximately $10^{-14}$ of full power flux
Why is such a small flux important?

- At first start up of a CANDU, this is the only flux present. It gets the thing started.
- After a very long shut down for maintenance this may be the largest source.
- A small source becomes significant because of **SUBCRITICAL MULTIPLICATION**.
- The core is configured to maximize neutron efficiency, so some of these source neutrons cause induced fissions.

Subcritical Multiplication

- Subcritical Multiplication varies, depending on how deeply “shut down” the core is
- Even in “Guaranteed Shut-down State” (GSS), with hundreds of mk of absorber added to the core, a few source neutrons cause fissions.
- As absorber is removed, bringing the reactor closer to critical, the number of fissions caused by source neutrons increases.
- The source strength doesn’t change, but fewer neutrons are absorbed uselessly.
How is this different than a Critical Core?

- A single (one time) pulse of neutrons injected into a critical core is self sustaining:
  - there are just as many neutrons in the next generation, i.e. $k = 1$
- A single (one time) pulse of neutrons injected into a sub-critical core ($k < 1$) produces fewer neutrons in each successive generation.
- the pulse will die out, but,
  - notice that $k \neq 0$: there are some fissions
  - neutrons in successive generations come from fission, not from the source.

So why doesn’t the flux die out with $k < 1$?

- Each “pulse” does die out, but the source continually injects new “pulses”.
- There are successive generations of neutrons from fissions, induced by some of the source neutrons, so the measured flux is always larger than the source flux.
- The observed (measured) flux depends on how close the reactor is to critical:
  - deeply subcritical: each generation drops quickly
  - almost critical: many neutrons survive
A Neutron Amplifier!

- The configuration of fuel, moderator, reflector and neutron absorber contributes to the number of induced fissions.
- The observed flux is always higher than the source flux, so we can think of the subcritical reactor as a neutron amplifier.
- The source flux is the input signal.
- The observed flux is the output.
- The gain, as we shall show, is \( \frac{1}{1 - k} \).

The Photo-neutron Source

The Most Important CANDU Source

The size of this source is about 0.03% of full power flux.
Photo-neutrons

- Some energetic $\gamma$-ray from ($\beta^{-},\gamma$) decay of fission products (and, perhaps, activation products) interact with deuterium nuclei in the heavy water moderator and coolant, ejecting the neutrons.
- The H-2 binding energy is 2.2 MeV, so the $\gamma$-rays must have energy in excess of this.

Delayed (fission) Neutrons

- These are not really source neutrons, but they provide a useful comparison
- Some fission product ($\beta^{-},\gamma$) decays produce excited daughter nuclei that emit an energetic neutron when they decay.
- Photo-neutrons and delayed neutrons both depend on the presence of fission products.
  - they are indirectly caused by fission
  - D.N.s are about 0.5% of the fission neutrons.
  - P.N.s strength is lower by more than an order of magnitude
How do d.n. and p.n Differ?

- The delayed neutrons show up within seconds or minutes of the fission that caused them. They affect the dynamics of power change, but not long term behaviour.
- Some photo-neutrons also show up within seconds, but many do not appear for days.
  - photoneutrons are produced in the shutdown core from fission products from previous high power operation, even months after shutdown

The Longest Lived Source

- The longest lived photo-neutrons so far positively identified come following the decay of Ba-140, with a decay half life of 12.8 days. After 3 months this source strength drops by a factor \((1/2)^{91/12.8} = 0.007\)
- Data from CANDU reactors with long shut down times suggest an additional longer lived source at very low concentration.
Relative Strength

- Suppose the full power flux in the fuel is about $1 \times 10^{14}$ n cm$^{-2}$ s$^{-1}$
- This is made up, approximately, as follows:
  - prompt fission neutrons $99,434,000,000,000$
  - delayed fission neutrons $534,000,000,000$
  - photo-neutrons $32,000,000,000$
  - spontaneous fission neutrons $1$
  - TOTAL $100,000,000,000,000$
- last digits uncertain. Actual values depend on fuel burnup

Subcritical Multiplication

$k < 1$

- In a subcritical core with sources, at any instant there are present in the core:
  - the source neutrons injected in this generation
  - the fission neutrons from the source neutrons in the previous generation ($\text{source} \times k$)
  - fission neutrons from fission neutrons from the source injected two generations ago ($\text{source} \times k^2$)
  - and all fission neutrons left from previous generations
Amplification

- Adding together all the neutrons from this generation, and all those produced by previous generations:
  \[ \phi_{\text{observed}} = \phi_{\text{source}} + k\phi_{\text{source}} + k \times (k\phi_{\text{source}}) + \text{etc.} \]

- We saw this arithmetic in chapter 8 and we know the result is
  \[ \phi_{\text{observed}} = \phi_{\text{source}} (1 + k + k^2 + k^3 + \ldots) \]

- This graph shows the dramatic increase in source multiplication close to critical.
Change in Power Level

- When $k$ is increased in a subcritical reactor the source stays the same, but the number of fissions increases, so flux rises.

\[
\phi_{\text{initial}} = \left( \frac{1}{1 - k_i} \right) \cdot \phi_{\text{source}}
\]

\[
\phi_{\text{final}} = \left( \frac{1}{1 - k_f} \right) \cdot \phi_{\text{source}}
\]

- The equation without $\phi_{\text{source}}$ is useful, because the source is usually not known.

Measurement of Core Reactivity

- Add a known amount of reactivity.
  - The difference $(k_f - k_i)$ is known.
    - e.g. A 15% decrease in liquid zone levels is worth 1 mk (typical: giving 100% worth 6.7 mk)
    - $(k_f - k_i) = 0.001$
    - Power is measured before and after
  - The flux ratio = the power ratio
  - The equation can be solved for $k$
Measurement of Device Reactivity Worth

- Core reactivity (or $k_i$) can be measured, as in the preceding slide.
  All that is required is a method to change core reactivity by a known amount.
- Now, insert an unmeasured absorbing rod and measure power before and after.
  This is a measurement of rod worth.
- $k_f = k_i + (\text{rod worth})$

\[
\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \frac{1 - k_i}{1 - k_f}
\]

Dynamic Response

Subcritical
Geometric Series (again)!

- \( S_V = 1 + k + k^2 + k^3 + \ldots + \ldots = 1/(1 - k) \)
- \( S_n = (1 + k + k^2 + k^3 + \ldots + k^n) \)
- \( S_n = S_V - k^{n+1} S_V \)

\[
\phi_{\text{observed}} = \phi_{\text{source}} \geq \frac{1}{k^{n+1}} - \phi_{\text{source}} \geq \frac{k^{n+1}}{1 - k}
\]

- The last term is transient
- It “disappears” when \( k^n \) is small enough
  - for \( k = 0.5 \), \( k^n < 0.01 \) in 7 generations
    - \( 1/(1-k) = 2 \)
  - for \( k = 0.999 \), \( k^n < 0.01 \) in 4,600 generations
    - \( 1/(1-k) = 1000 \)

Step Reactivity Additions

- For Illustration
- Add +10 mk of reactivity
- Then add +10 mk more
- Continue
  \( k \) increasing each step

The reactor gets closer to critical with each step
- The rise gets bigger on each addition, and
- Time to reach new level gets longer
Critical & Subcritical

◆ Add a small $\Delta k$ step:
  - When subcritical, but very close to critical, there is a large power rise that flattens out after a long time
  - When slightly supercritical, the initial response is almost the same, but eventually the power begins to rise exponentially
    • and can reach any level if not stopped
◆ Chapter 14 will look at the transition from subcritical to critical

What is Critical?

◆ Theoretically, criticality is $k = 1.000\ 000 \equiv$
  - this is “unmeasureable” to sufficient accuracy to be a practical definition
◆ Operationally, the reactor is “critical” whenever it is close enough to critical for the regulating system to maneuver power to a requested level at the requested rate
◆ called “direct regulating system control”
◆ Station operating rules may allow
  e.g. 5% - 10% zone level subcritical operation
  - this is typically about $\Delta k = -0.3\ mk$ to $-0.7\ mk$
How Long does the Power Rise take?

- We can estimate \( n \) from \( k^n < 0.01 \) but to get \( n \) we need the time for one neutron generation.
- An average value is not good enough.
- We must wait for the longer lived delayed neutrons to reach equilibrium.
- A reasonable guess is to wait for the group 2 delayed neutrons to reach equilibrium.
  - see next slide (\( L \) \( \approx 0.995 \) \( l + 0.005 \times 30 \) s = 0.15 s)
- Pick an neutron lifetime around 0.1 s or 0.2 s

### 6 Groups of Delayed Neutrons
Data for an Equilibrium Fuelled CANDU

<table>
<thead>
<tr>
<th>GROUP</th>
<th>DELAYED NEUTRON FRACTION % OF TOTAL OF ( \beta ) = 0.54 %</th>
<th>HALF LIFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.5%</td>
<td>0.2s</td>
</tr>
<tr>
<td>5</td>
<td>14.0%</td>
<td>0.5s</td>
</tr>
<tr>
<td>4</td>
<td>39.0%</td>
<td>2.2s</td>
</tr>
<tr>
<td>3</td>
<td>19.0%</td>
<td>5.7s</td>
</tr>
<tr>
<td>2</td>
<td>21.0%</td>
<td>22.0s</td>
</tr>
<tr>
<td>1</td>
<td>3.5%</td>
<td>54.2 s</td>
</tr>
</tbody>
</table>
CANDU Photo-neutron data

<table>
<thead>
<tr>
<th>GROUP</th>
<th>PHOTONEUTRON FRACTION % OF TOTAL OF PN ~ 0.033 %</th>
<th>HALF LIFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>64.6%</td>
<td>2.5s</td>
</tr>
<tr>
<td>14</td>
<td>20.3%</td>
<td>41.0s</td>
</tr>
<tr>
<td>13</td>
<td>7.0%</td>
<td>2.40m</td>
</tr>
<tr>
<td>12</td>
<td>3.3%</td>
<td>7.70m</td>
</tr>
<tr>
<td>11</td>
<td>2.1%</td>
<td>27.0m</td>
</tr>
<tr>
<td>10</td>
<td>2.3%</td>
<td>1.65h</td>
</tr>
<tr>
<td>9</td>
<td>0.3%</td>
<td>4.41h</td>
</tr>
<tr>
<td>8</td>
<td>0.1%</td>
<td>53.04h</td>
</tr>
<tr>
<td>7</td>
<td>0.05%</td>
<td>12.815d</td>
</tr>
</tbody>
</table>

Note: The longest lived photo-neutron source is due mainly to the decay of the fission product Ba-140, for which about 4% of β decays yield γ-rays with E > 2.23 MeV.

The Two Group Equation for the Subcritical Core

- A constant source term can be added to the differential equations of Chapter 8
  - so a time dependent solution can be derived for subcritical power changes
- We will not do this.
- Instead we quote the formula, simplified to the “prompt jump” approximation
- And show some plotted graphs
Dynamics Equation (simplified) for the Subcritical Core

- Here, \(-\rho \downarrow 1 - k\) in the notation of our text
  - not very accurate if \(k \ll 1\): \(\rho = (k - 1)/k\) is better

\[
P(t) \approx \frac{P_{source}}{-\rho_f} \left[ 1 - \frac{\rho_f}{\rho_i} \frac{t}{e^\tau} \right]
\]

- Use the group 2 decay constant \(\lambda = 0.03\) s\(^{-1}\)
  - formula assumes all delayed neutrons have the same \(\lambda\).

Response of a Deeply Subcritical Core to a Large Reactivity Addition

SDS#1 Repository (+30 mk): Initially 400 mk Subcr

12% rise in a small fraction of a second (5 ms)

Initial Power Level may be about \(10^{-7}\) or so.
It rises to about \(10^{-6.95}\).
Response of an Almost Critical Core to a Small Reactivity Addition

5% Zone Level Drop (+0.3 mk): Initially 1.5 mk Subcri

About 5 minutes to Stabilize at 25% above the initial power

On a typical restart, after a maintenance shutdown of a few weeks, initial power might be $10^{-4}$. It increases to $10^{-3.9}$

Notice the Prompt Jump, followed by a Stable Rise