Dynamics of the Subcritical Core

Neutron Sources
Subcritical Multiplication of Sources
Response Time to +Δk

What do Sources Do?

- There is measurable flux in a shut down reactor core, even if the reactor in the Guaranteed Shutdown State (GSS), with very large amounts of neutron absorbing stuff in the core.
- This is caused by subcritical multiplication of source neutrons.

Why is there Flux in a Shut Down Reactor? How is it possible to get the first fission?

- A REACTOR, ONCE STARTED, CANNOT BE SHUT DOWN
- We refer to the reactor as:
  - in the operating state
  - in the shut down state
- But the Shut Down Reactor Core is not de-energized like other machines:
  - there is heat from fission product decay
  - there is flux from subcritical *source* multiplication

What is a Neutron Source?

- A neutron source injects a steady supply of neutrons, independent of power level, temperature, or any controllable parameter.
- CANDU reactors have two important natural sources:
  - spontaneous fission of U-238;
  - photo-neutrons released from deuterium nuclei in the heavy water by energetic γ -rays from fission product decay.

Spontaneous Fission

We will look at this source first in
some detail.

The size of this source is about 10^{-14}
of full power flux; ♠ 1 n⊇cm^{-2} s^{-1}.

Spontaneous Fission of U-238

- Each CANDU fuel bundle has, initially, about 18.9 kg of natural uranium, 99.3% U-238.
- A CANDU-6 has 380 fuel channels & 12 bundles in each channel, over 85 Mg of U-238.
- This large quantity overwhelms any spontaneous fissions of other isotopes in the fuel.
  - Compare the U-235 content of fresh fuel (0.7%)
  - This is just over 600 kg of fissile fuel in a fresh core.

Spontaneous Fission Source Strength

- S.F. decay of U-238 is a very rare process.
- The Half-Life is $T_{1/2} = 8 \text{ x } 10^{15}$ years
- This results in over 1/2 million decays per second in the whole core, but,
- The neutrons are quickly absorbed, in about a milli-second, so
- At any instant in the core there are just over 1000 neutrons from spontaneous fission.

Neutron Density & Neutron Flux

- The Volume of the CANDU core, a cylinder about 6 m long, diameter 7 m, is over 200 m^3
- 1000 neutrons is 5 neutrons per cubic meter!
  - Most neutrons slow down before they are absorbed
- Thermal neutron speed is about 2200 m/s
- Flux (ϕ) is (neutron density x speed)

$$\phi = 5 \text{ x } 2200 \text{ x } 10^{-4} \spadesuit 1 \text{ cm}^{-2} \text{ s}^{-1}$$

This is approximately 10^{-14} of full power flux

Why is such a small flux important?

- At first start up of a CANDU this is the only flux present. It gets the thing started.
- After a very long shut down for maintenance this may be the largest source.
- A small source becomes significant because of **SUBCRITICAL MULTIPLICATION**.
- The core is configured to maximize neutron efficiency, so some of these source neutrons cause induced fissions.

Subcritical Multiplication

- Subcritical Multiplication varies, depending on how deeply “shut down” the core is
- Even in “Guaranteed Shut-down State” (GSS), with hundreds of mk of absorber added to the core, a few source neutrons cause fissions.
- As absorber is removed, bringing the reactor closer to critical, the number of fissions caused by source neutrons increases.
- The source strength doesn’t change, but fewer neutrons are absorbed uselessly.

How is this different than a Critical Core?

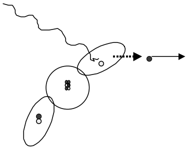
- A single (one time) pulse of neutrons injected into a critical core is self sustaining:
  - there are just as many neutrons in the next generation, i.e. k = 1
- A single (one time) pulse of neutrons injected into a sub-critical core (k < 1) produces fewer neutrons in each successive generation.
- the pulse will die out, but,
  - notice that k ℵ 0: there are some fissions
  - neutrons in successive generations come from fission, not from the source.

So why doesn't the flux die out with k < 1?

- Each "pulse" does die out, but the source continually injects new "pulses".
- There are successive generations of neutrons from fissions, induced by some of the source neutrons, so the measured flux is always larger than the source flux.
- The observed (measured) flux depends on how close the reactor is to critical:
  - deeply subcritical: each generation drops quickly
  - almost critical: many neutrons survive

A Neutron Amplifier!

- The configuration of fuel, moderator, reflector and neutron absorber contributes to the number of induced fissions.
- The observed flux is always higher than the source flux, so we can think of the subcritical reactor as a neutron amplifier
- The source flux is the input signal
- The observed flux is the output
- The gain, as we shall show, is $\boxed{\frac{1}{1-k}}$



The Photo-neutron Source

The Most Important CANDU Source

$${}_{1}^{2}H\left(\gamma\,,n\right){}_{1}^{1}H$$

The size of this source is about 0.03% of full power flux

Photo-neutrons

- Some energetic γ-ray from (β^-,γ) decay of fission products (and, perhaps, activation products) interact with deuterium nuclei in the heavy water moderator and coolant, ejecting the neutrons.
- The H-2 binding energy is 2.2 MeV, so the γ-rays must have energy in excess of this.

Delayed (fission) Neutrons

- These are not really source neutrons, but they provide a useful comparison
- Some fission product (β-,γ) decays produce excited daughter nuclei that emit an energetic neutron when they decay.
- Photo-neutrons and delayed neutrons both depend on the presence of fission products.
  - they are indirectly caused by fission
  - D.N.s are about 0.5% of the fission neutrons.
  - P.N.s strength is lower by more than an order of magnitude

How do d.n. and p.n Differ?

- The delayed neutrons show up within seconds or minutes of the fission that caused them. They affect the dynamics of power change, but not long term behaviour.
- Some photo-neutrons also show up within seconds, but many do not appear for days.
  - photoneutrons are produced in the shutdown core from fission products from previous high power operation, even months after shutdown

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The Longest Lived Source

- The longest lived photo-neutrons so far positively identified come following the decay of Ba-140, with a decay half life of 12.8 days. After 3 months this source strength drops by a factor $(1/2)^{91/12.8} = 0.007$
- Data from CANDU reactors with long shut down times suggest an additional longer lived source at very low concentration.

Relative Strength

- Suppose the full power flux in the fuel is about 1 x 10^{14} n cm^{-2} s^{-1}
- This is made up, approximately, as follows

prompt fission neutrons	99,434,000,000,000
delayed fission neutrons	534,000,000,000
photo-neutrons	32,000,000,000
spontaneous fission neutrons	1
TOTAL	100,000,000,000,000

- last digits uncertain. Actual values depend on fuel burnup

Subcritical Multiplication
k < 1

- In a subcritical core with sources, at any instant there are present in the core:
  - the source neutrons injected in this generation
  - the fission neutrons from the source neutrons in the previous generation (source x k)
  - fission neutrons from fission neutrons from the source injected two generations ago (source x k^2)
    - and all fission neutrons left from previous generations

Amplification

- Adding together all the neutrons from this generation, and all those produced by previous generations:

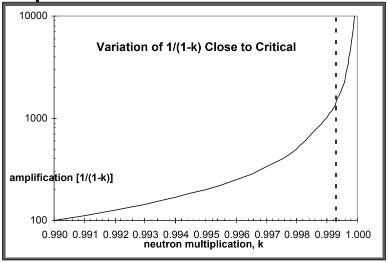
$\phi_{observed} = \phi_{source} + k\phi_{source} + k \times (k\phi_{source}) + \text{etc.}$

$\phi_{observed} = \phi_{source}\,(1 + k + k^2 + k^3 + \ldots)$

- We saw this arithmetic in chapter 8 and we know the result is

$$\boxed{\phi_{observed} = \left(\frac{1}{1-k}\right)\cdot\phi_{source}}$$

Amplification Close to Critical



- This graph shows the dramatic increase in source multiplication close to critical.

Change in Power Level

- When k is increased in a subcritical reactor the source stays the same, but the number of fissions increases, so flux rises.

$$\phi_{\text{initial}} = \left(\frac{1}{1-k_i}\right)\cdot\phi_{\text{source}}$$

$$\phi_{\text{final}} = \left(\frac{1}{1-k_f}\right)\cdot\phi_{\text{source}}$$

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1-k_i}{1-k_f}\right)$$

- The equation without ϕ_{source} is useful, because the source is usually not known.

Measurement of Core Reactivity

- Add a known amount of reactivity.

  The difference $(k_f - k_i)$ is known.
  - e.g. A 15% decrease in liquid zone levels is worth 1 mk (typical: giving 100% worth 6.7 mk)
  - $(k_f - k_i) = 0.001$
  - Power is measured before and after

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1-k_i}{1-k_f}\right)$$

- The flux ratio = the power ratio
- The equation can be solved for k

Measurement of Device Reactivity Worth

- Core reactivity (or k_i) can be measured, as in the preceding slide

  All that is required is a method to change core reactivity by a known amount.

$$\frac{\phi_{\text{final}}}{\phi_{\text{initial}}} = \left(\frac{1-k_i}{1-k_f}\right)$$

- Now, insert an unmeasured absorbing rod and measure power before and after.

  This is a measurement of rod worth.
- $k_f = k_i + (\text{rod worth})$

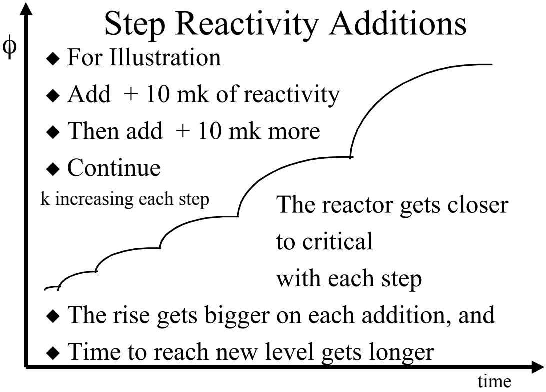
Dynamic Response

Subcritical

Geometric Series (again)!

- $S_{\forall} = 1 + k + k^2 + k^3 + \ldots + \ldots = 1/(1 - k)$
- $S_n = (1 + k + k^2 + k^3 + \ldots + k^n)$
- $S_n = S_{\forall} - k^{n+1} S_{\forall}$
- $\boxed{\phi_{observed} = \phi_{source} \supseteq \left(\frac{1}{1-k}\right) - \phi_{source} \supseteq k^{n+1} \left(\frac{1}{1-k}\right)}$
- The last term is transient
- It “disappears” when k^n is small enough
  - for $k = 0.5$, $k^n < 0.01$ in n ♠ 7 generations
    - $1/(1-k) = 2$
  - for $k = 0.999$ $k^n < 0.01$ in n ♠ 4 600 generations
    - $1/(1-k) = 1000$

Step Reactivity Additions



- For Illustration
- Add + 10 mk of reactivity
- Then add + 10 mk more
- Continue

k increasing each step

The reactor gets closer
to critical
with each step

- The rise gets bigger on each addition, and
- Time to reach new level gets longer

Critical & Subcritical

- Add a small Δk step:
  - When subcritical, but very close to critical, there is a large power rise that flattens out after a long time
  - When slightly supercritical, the initial response is almost the same, but eventually the power begins to rise exponentially
    - and can reach any level if not stopped
- Chapter 14 will look at the transition from subcritical to critical

What is Critical?

- Theoretically, criticality is k = 1.000 000 ≡
  - this is “unmeasureable” to sufficient accuracy to be a practical definition
- Operationally, the reactor is “critical” whenever it is close enough to critical for the regulating system to maneuver power to a requested level at the requested rate
- called “*direct regulating system control*”
- Station operating rules may allow

  e.g. 5% - 10% zone level subcritical operation
  - this is typically about Δk = - 0.3 mk to - 0.7 mk

How Long does the Power Rise take?

- We can estimate n from $k^n < 0.01$ but to get n we need the time for one neutron generation.
- An average value is not good enough
- We must wait for the longer lived delayed neutrons to reach equilibrium
- A reasonable guess is to wait for the group 2 delayed neutrons to reach equilibrium
  - see next slide (L ♠ 0.995 l + 0.005 x 30 s = 0.15 s)
- Pick an neutron lifetime around 0.1 s or 0.2 s

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6 Groups of Delayed Neutrons Data for an Equilibrium Fuelled CANDU

GROUP	DELAYED NEUTRON FRACTION % OF TOTAL OF β = 0.54 %	HALF LIFE weightedav = 8.4 s mean lifetime 12.2 s
6	3.5%	0.2s
5	14.0%	0.5s
4	39.0%	2.2s
3	19.0%	5.7s
2	21.0%	22.0s
1	3.5%	54.2 s

CANDU Photo-neutron data

GROUP	PHOTONEUTRON FRACTION % OF TOTAL OF $\beta_{PN} \approx$ 0.033 % n)	HALF LIFE
15	64.6%	2.5s
14	20.3%	41.0s
13	7.0%	2.40m
12	3.3%	7.70m
11	2.1%	27.0m
10	2.3%	1.65h
9	0.3%	4.41h
8	0.1%	53.04h
7	0.05%	12.815d

Note: The longest lived photo-neutron source is due mainly to the decay of the fission pr
Ba-140, for which about 4?% of β decays yield γ-rays with E > 2.23 MeV.

The Two Group Equation for the Subcritical Core

- A constant source term can be added to the differential equations of Chapter 8
  - so a time dependent solution can be derived for subcritical power changes
- We will not do this.
- Instead we quote the formula, simplified to the "prompt jump" approximation
- And show some plotted graphs

Dynamics Equation (simplified) for the Subcritical Core

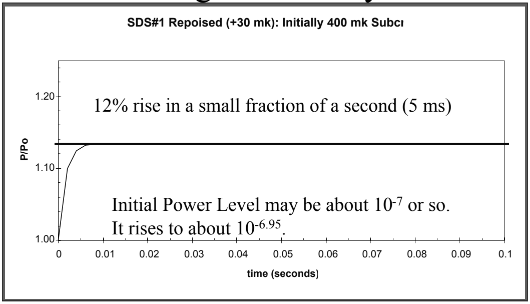
- Here, - ρ ♠ 1 - k in the notation of our text
  - not very accurate if k << 1: $\rho = (k - 1)/k$ is better

$$P(t) \approx \frac{P_{source}}{\left(-\rho_f\right)}\left[1-\frac{\left(1-\frac{\rho_f}{\rho_i}\right)}{\left(1-\frac{\rho_f}{\beta}\right)}e^{\frac{t}{\tau}}\right]$$

Note that τ is negative

- Use the group 2 decay constant $\lambda = 0.03\ s^{-1}$
  - formula assumes all delayed neutrons have the same λ

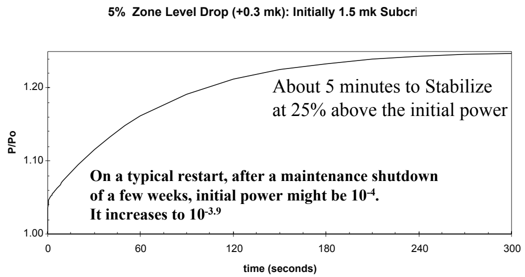
Response of a Deeply Subcritical Core to a Large Reactivity Addition



12% rise in a small fraction of a second (5 ms)

Initial Power Level may be about 10^{-7} or so.
It rises to about $10^{-6.95}$.

Response of an Almost Critical Core to a Small Reactivity Addition



5% Zone Level Drop (+0.3 mk): Initially 1.5 mk Subcri

About 5 minutes to Stabilize
at 25% above the initial power

On a typical restart, after a maintenance shutdown
of a few weeks, initial power might be 10^{-4}.
It increases to $10^{-3.9}$

Notice the Prompt Jump, followed by a Stable Rise