CHE 3804 NUCLEAR ENGINEERING

SECTION 4

REACTOR THEORY

ONE GROUP REACTOR EQUATION

Previously the equation for the diffusion of monoenergetic neutrons in a medium was derived. This *one group steady state diffusion equation* is given by:

$$D \nabla^2 \phi - \Sigma_a \phi = - s$$

The first term is related to the diffusion of neutrons in the medium while the second term on the left hand side is related to the absorption of neutrons in the medium. The term on the right hand side is the source term related to the production of neutrons.

In a critical reactor, where the number of neutrons in each succeeding generation is constant, that is k is equal to unity, the number of neutrons absorbed is equal to the number produced.

$$s = \eta \, \Sigma_{a \, fuel} \, \phi$$

Here η is the number of neutrons emitted per neutron absorbed in the fuel. A further factor f may be defined as the ratio of neutrons absorbed in the fuel to those absorbed in whole reactor.

$$f = \Sigma_{a \, fuel} / \Sigma_{a \, reactor}$$

This factor f is known as the fuel utilization factor. Substituting this into the above equation gives

$$s = \eta \, f \Sigma_{a \, reactor} \, \phi$$

For an infinite reactor with no surface from which neutrons can leak k_∞ is used to define the *number of neutrons in the current generation* over the *number of neutrons in the previous generation*. Thus k_∞ may be defined as follows:

$$k_\infty = \eta \, f \, (\Sigma_{a \, reactor} \, \phi) \, / \, (\Sigma_{a \, reactor} \, \phi)$$

$$k_\infty = \eta \, f$$

The source term may thus be redefined in terms of k_∞

$$s = k_\infty \, \Sigma_{a \, reactor} \, \phi$$

Substituting this into the one group diffusion equation gives:

$$D \, \nabla^2 \phi - \Sigma_{a \, reactor} \, \phi = - k_\infty \, \Sigma_{a \, reactor} \, \phi$$

This may then be rearranged to give the following equation:

$$\nabla^2 \phi + (k_\infty - 1) \, (\Sigma_{a \, reactor} / \, D) \, \phi = 0$$

The diffusion length L may be incorporated since:

$$\Sigma_a / \, D = L^2$$

Furthermore a new term B^2 known as the Buckling may be incorporated. Buckling is defined as follows:

$$B^2 = (k_\infty - 1) / \, L^2$$

The one group steady state diffusion equation then reduces to the one group reactor equation

$$\nabla^2 \phi + B^2 \, \phi = 0$$

The first term is related to the diffusion of neutrons while the second term is related to the production and absorption of neutrons.

REACTOR EQUATION APPLICATIONS

The one group reactor equation may be applied to a variety of different geometries. Applications requiring solutions in three dimensions are quite complex so a one dimensional application only will be considered. Similar principles apply to all geometries.

The Infinite Slab Reactor

For an infinite slab of thickness a the one group reactor equation simplifies as follows:

$$\nabla^2 \phi + B^2 \phi = 0$$

$$d^2\phi/dx + B^2 \phi = 0$$

If x is measured from the centre of the slab there is symmetry in the solution and boundary conditions may be fixed as follows:

ϕ is zero at the surface
ϕ is maximum at the centre

$$\phi_{x=a/2} = 0$$

The solution to the above differential equation is:

$$\phi_x = A \cos B_x$$

To satisfy the boundary conditions:

$$B = \pi / a$$

$$\phi_x = A \cos (\pi x / a)$$

The buckling B^2 is given by

$$B^2 = (\pi / a)^2$$

The constant A may be solved by considering the total power produced in the reactor. This is obtained by multiplying the macroscopic fission cross section by the neutron flux, integrating over the whole reactor and applying a conversion factor for the amount of energy produced in fission.

$$P = E_{conversion\,factor} \int_{-a/2}^{a/2} \Sigma_f \phi_x \, dx$$

$$E_R \, \Sigma_f \int_{-a/2}^{a/2} A \cos(\pi x/a) \, dx$$

$$= 2 \, a \, E_R \, \Sigma_f \, A \, / \, \pi$$

$$A = \pi \, P \, / \, 2 \, a \, \Sigma_R \, \Sigma_f$$

The complete solution for the infinite slab reaction is thus

$$\phi_x = (\pi \, P \, / \, 2 \, a \, \Sigma_R \, \Sigma_f) \cos(\pi x \, / \, a)$$

$$\phi_x = (1.57 \, P \, / \, a \, \Sigma_R \, \Sigma_f) \cos(\pi x \, / \, a)$$

REACTOR EQUATION SOLUTIONS

The practical reactor shapes are the rectangular parallelepiped, the finite cylinder and the sphere. By following mathematical procedures similar to that above, equations for the neutron flux at any point in the reactor may be obtained. The results are as follows:

Rectangular Reactor

$$\phi = (3.87 \, P \, / \, E_R \, \Sigma_f \, V) \cos(\pi x \, / \, a) \cos(\pi y \, / \, b) \cos(\pi z \, / \, c)$$

Cylindrical Reactor

$$\phi = (3.63 \, P \, / \, E_R \, \Sigma_f \, V) \, J_0 \, (2.405 \, r \, / \, R) \cos(\pi z \, / \, H)$$

Spherical Reactor

$$\phi = (0.25 \, P \, / \, E_R \, \Sigma_f \, R^2) \, (1 \, / \, r) \sin(\pi r \, / \, R)$$

In the above J_0 is a Bessel Function which may be obtained from tables.

NEUTRON FLUX AND POWER

Using the above solutions of the reactor equation the relationship between maximum flux and average flux for different reactor shapes may be obtained. This is important in the design of a reactor since the power produced is proportional to the neutron flux. The maximum power and maximum flux at any point is limited by the fuel design while the average power and average flux of the reactor determines the overall power output.

Spherical Reactor

The maximum flux occurs in the centre of the reactor that is when the radius is equal to zero.

$$\phi = (0.25 \, P \, / \, E_R \, \Sigma_f \, R^2) \, (1 \, / \, r) \sin (\pi r \, / \, R)$$

$$\phi_{max} = (P \, / \, 4 \, E_R \, \Sigma_f \, R^2) \, \text{Limit}_{r \to 0} \, \{ \sin (\pi r \, / \, R) \, / \, r \}$$

$$\phi_{max} = \pi \, P \, / \, 4 \, E_R \, \Sigma_f \, R^2 \, R$$

The average flux is given by the flux in any elemental volume integrated over the whole volume and divided by the total volume

$$\phi_{average} = \int \phi \, dV \, / \, V$$

The total reactor power is obtained by multiplying the macroscopic fission cross section by the neutron flux, integrating over the whole reactor and applying a conversion factor for the amount of energy produced in fission

$$P = E_{conversion \, factor} \int \Sigma_f \, \phi \, dV$$

$$\int \phi \, dV = P \, / \, E_R \, \Sigma_f$$

If this is substituted into the above equation an expression for the average flux is obtained

$$\phi_{average} = P \, / \, E_R \, \Sigma_f \, V$$

The ratio of maximum flux to average flux may then be obtained

$$\phi_{max} \, / \, \phi_{ave} = (\pi \, P \, / \, 4 \, E_R \, \Sigma_f \, R^2 \, R) \, / \, (P \, / \, E_R \, \Sigma_f \, V)$$

$$= (\pi / 4 \, R^3)(4\pi \, R^3 / 3)$$

$$= 3.29$$

MAXIMUM TO AVERAGE FLUX

The ratios of maximum to average flux may be determined for different reactor shapes and are given below

Rectangular Reactor

$$\phi_{max} / \phi_{ave} = 3.88$$

Spherical Reactor

$$\phi_{max} / \phi_{ave} = 3.29$$

Cylindrical Reactor

$$\phi_{max} / \phi_{ave} = 3.64$$

ONE GROUP CRITICAL EQUATION

From the one group reactor equation already derived the buckling may be expressed in terms of the neutron multiplication factor k_{∞} for an infinite reactor:

$$\nabla^2 \phi + B^2 \phi = 0$$

$$B^2 = (k_{\infty} - 1) / L^2$$

This is the condition for a reactor in the steady state or critical condition. The buckling B^2 depends upon the dimensions and geometry of the reactor while k_{∞} and L depend upon the physical properties of the reactor. Thus to create a critical condition in a reactor one or other side of the equation must be adjusted to become equal to the other side. If the physical properties of the reactor are given then, to obtain a critical condition the dimensions of it must be selected appropriately. Alternatively if the reactor geometry is known then the physical properties need to be adjusted to obtain criticality.

When considering a reactor of given geometry and hence finite dimensions there is inevitably a certain leakage of neutrons from its surface. The degree of leakage or absorption may be determined by comparing the number of neutrons leaking from the system with the number of neutrons absorbed in the system. The number leaking from the system is the flow of neutrons across an elemental surface integrated over the whole surface.

$$\text{Number leaking} = \int_A J \, n \, d \, A$$

$$= \int_V \text{div} \, J \, dV$$

$$= - D \int_v \nabla^2 \phi \, dV$$

This may be transformed using the one group reactor equation.

$$\text{Number leaking} = D \int_v B^2 \, \phi \, dV$$

The number absorbed in the system is given by the macroscopic absorption cross-section and the flux integrated over the whole volume.

$$\text{Number absorbed} = \Sigma_a \int_v \phi \, dV$$

The probability of absorption or non-leakage P_L is the ratio of the number absorbed over the total number absorbed and leaking.

$$P_L = \Sigma_a \int_v \phi \, dV \, / \, [\Sigma_a \int_v \phi \, dv + D \, B^2 \int_v \phi \, dv]$$

$$= \Sigma_a \, / \, (\Sigma_a + D \, B^2)$$

$$= 1 \, / \, (1 + B^2 \, L^2)$$

The one group reactor equation may be rewritten to isolate the neutron multiplication factor k_∞:

$$k_\infty = 1 + B^2 \, L^2$$

If this is substituted into the equation for the probability of non-leakage then:

$$P_L = 1 \, / \, k_\infty$$

$$k_{\infty} P_L = 1$$

In a critical system the number of neutrons absorbed is:

$$\text{Number absorbed} = \Sigma_a \int_v \phi \, dV$$

This leads to the number of neutrons released:

$$\text{Number released} = \eta \, f \, \Sigma_a \int_v \phi \, dV$$

$$= k_{\infty} \Sigma_a \int_v \phi \, dV$$

Of this number a certain proportion are lost by leakage. The number remaining on not leaking from the system is given by:

$$\text{Number not leaking: } P_L \, k_{\infty} \, \Sigma \int_v \phi \, dV$$

This is the number in the next generation of neutrons. From the definition of the neutron multiplication factor k the following is obtained

$$k = P_L \, k_{\infty} \, \Sigma_a \int_v \phi \, dV \, / \, \Sigma_a \int_v \phi \, dV$$

$$k = k_{\infty} \, P_L$$

Here k is the neutron multiplication factor for a finite reactor and k_{∞} the neutron multiplication factor for an infinite reactor. P_L is the probability on non-leakage through the surface of the reactor.

THE FOUR FACTOR FORMULA

So far consideration has been given only to mono-energetic neutrons presumed to have thermalised immediately after having been produced. On this assumption the neutron multiplication factor k is given by two factors namely the number of neutrons emitted per neutron absorbed η and the thermal utilization factor f

$$k = \eta \, f$$

$$\eta = \nu \, \Sigma_{f \text{ fuel}} \, / \, \Sigma_{a \text{ fuel}}$$

$$f = \Sigma_{a\ fuel} / \Sigma_{a\ reactor}$$

In a reactor the neutrons produced by fission at high energy must be moderated to reduce their energies. If the moderation occurs entirely within the moderator the above factors give a satisfactory result. In reality however some high energy neutrons cause fission in U-238 before entering the moderator. This creates a few additional neutrons not previously anticipated. Furthermore some neutrons may interact with the fuel while still in the intermediate or epithermal energy range and be captured by the U-238 resonances. Two additional factors are therefore required. The fast fission factor ϵ takes account of the additional neutrons produced by the fissioning of U-238 by fast neutrons. This factor is slightly greater than unity. The resonance escape probability p is the probability of not being absorbed in the resonances of U-238 while slowing down to thermal conditions. This factor is less than unity since some are always absorbed. These four factors may be summarised as follows:

$$k_\infty = p\ \epsilon\ \eta\ f$$

f = Thermal utilization factor
η = Neutrons emitted per neutron absorbed
ϵ = Fast fission factor
p = Resonance escape probability

THE SIX FACTOR FORMULA

The four factor formula applies to an infinite reactor with no free surface from which neutrons may be lost. In a real reactor leakage takes place such that k is less than k_∞ by the non-leakage probability P_L

It is convenient to divide the non-leakage factor into two separate factors to take account of fast neutron leakage and thermal neutron leakage. Thus two new factors namely the fast neutron non-leakage probability Λ_f and the thermal neutron non-leakage probability Λ_t are obtained

$$k = k_\infty\ \Lambda_f\ \Lambda_t$$

$$k = p\ \epsilon\ \eta\ f\ \Lambda_f\ \Lambda_t$$

This is the six factor formula for neutron multiplication.

NEUTRON CYCLE

The four or six factor formula may be applied to the neutron cycle. In each phase the appropriate factor may be applied and the total number of neutrons tracked through the cycle. In the algebraic example given the cycle starts with N thermal neutrons absorbed and ends with $N \epsilon p \eta f \Lambda_f \Lambda$ neutrons available to be absorbed in the next generation giving a multiplication of $\epsilon p \eta f \Lambda_f \Lambda_t$. From the definition of the neutron multiplication factor this is equal to k. In the numerical example given the cycle starts with 1000 fast neutrons being produced from fission and ends with 1000 being produced in the next generation. Note that the two examples start at different points in the cycle but this is of little consequence.

REACTOR SHAPES

The non-leakage probability P_L varies with shape and size of the reactor. This is due to the differing surface to volume ratio in different shapes of the same volume and a changing surface to volume ratio in different sizes of the same shape. For the most common shapes a rectangular block has a greater surface to volume ratio than a cylinder and a cylinder a greater surface to volume ratio than a sphere. Any particular shape will have a decreasing surface to volume ratio as the size or volume increases since the surface is proportional to the square of the linear dimension and the volume proportional to the cube of the same dimension. The smaller the surface to volume ratio the smaller the value of the non-leakage probability P_L.

NEUTRON LEAKAGE

At the boundary of a reactor neutrons leak from the surface such that the neutron multiplication factor k for an actual reactor is less than the neutron multiplication factor k_∞ for an infinite reactor. This loss of neutrons is detrimental to the overall performance of the reactor since they have come from the fissioning of fuel. Since unnecessary consumption of fuel is undesirable it is advantageous to try to reduce this leakage.

One way of reducing the leakage is of course to minimise the surface area of the reactor by selecting an appropriate geometry for the reactor. Further savings can be achieved by installing a reflector around the core. This does not actually reflect the neutrons as a mirror would reflect light. Rather it allows neutrons to diffuse far enough into the reflecting medium so that there is a significant probability that some will diffuse back into the core.

REACTOR REFLECTORS

The properties of a good reflector are the same as those of a good moderator. A reflector must have a high neutron scattering cross-section and a low neutron absorption cross-section. Obviously neutrons should not be absorbed in the reflector while a high probability of scattering ensures that the neutrons are soon deflected from their original path. After a number of random scattering collisions the neutrons could be travelling in any direction including back towards the reactor core. In a flat reflector of adequate thickness and with perfect neutron properties mounted on the side of a reactor having a plane surface half of the neutrons would leave the reflector on the inside to return to the reactor and half would leave on the outside to be lost forever. More loss would occur at a corner or edge so that a reflector can never be as good as 50% efficient. Nevertheless this represents a substantial saving in neutrons and nearly all reactors make use of a reflector. The reflector is invariably an extension of the moderator a short distance beyond the outermost fuel elements.

NEUTRON FLUX

The neutron flux at the boundary of a bare reactor (no reflector) falls to a low value at the surface. It is assumed to fall to zero a short distance beyond the surface at the extrapolation distance to permit the application of diffusion theory. This compensates for the surface effects and allows the neutron flux to be predicted using the diffusion equation.

With a reflected reactor neutrons diffuse through the reflector and some are returned to the reactor core. This has the effect of enhancing the neutron flux near the surface of the reactor core. Neutrons that would otherwise be lost are returned and added to the existing neutron flux just inside the boundary. Furthermore the neutron flux in the reflector may actually be greater than that in the core near the boundary since there is no fuel to absorb neutrons and the neutrons tend to accumulate while diffusing in the reflector. The neutron flux thus decreases with distance from the centre of the reactor but this decrease levels off and may reverse near the boundary. In the reflector the neutron flux actually rises with increasing distance from the centre but then falls to a very low value near the outer surface of the reflector.

FLUX FLATTENING

A reflector on a reactor has a further advantage besides improving the neutron utilization in the reactor. It increases the $\phi_{average}$ / $\phi_{maximum}$ ratio in the reactor. Since power is proportional to neutron flux this means that more power is obtained from the reactor without increasing

the maximum flux. Thus the highest rated fuel elements can still operate within the design power, and hence temperature, limitations. Flux flattening is highly desirable in all power reactors and other methods are also employed to obtain the most uniform neutron flux across the reactor as is possible.

FLUX FLATTENING IN CANDU REACTORS

CANDU reactors employ four methods of flux flattening to increase the ratio of $\phi_{average}$ / $\phi_{maximum}$.

Reflector

A reflector returns some neutrons to the reactor core so enhancing the neutron flux near the reactor surface. In a CANDU reactor this is done around the cylindrical core and thus has an effect in a radial direction only. Refuelling is done from the ends of the reactor so there is no space for a reflector of adequate thickness.

Adjuster Rods

Adjuster rods which absorb neutrons weakly are inserted into the centre of the reactor to reduce the neutron flux in that region. Although inserted from the top they can be designed to be most effective in the very centre of the reactor core. The effect is in all directions both axial and radial. Once the neutron flux in the centre has been suppressed the overall reactor power can be increased to bring the maximum neutron flux back up to its maximum allowable value. The average neutron flux is then higher than prior to the insertion of the adjuster rods since the flux profile has been changed into a flatter shape.

Bi-directional Fuelling

In a CANDU reactor the horizontal fuel channels each contain twelve fuel bundles. If only eight are removed by inserting eight new ones from the other end the effect is to skew the neutron flux towards the end having the fresh fuel. If the adjacent fuel channel is refuelled in the same way but from the opposite side the neutron flux will be skewed the other way. The net effect is a summation of the two flux profiles giving a flux profile which is somewhat flatter than the cosine shape that would occur if the whole of each fuel channel had been fuelled with new fuel. This effect naturally only occurs in the axial direction.

Differential Fuel Burnup

By dividing the reactor core fuel channels into an inner zone and an outer zone and allowing the fuel in the inner zone to burn up to a greater degree than that in the outer zone the neutron flux near the centre of the reactor can be suppressed. This allows the fuel channels in the outer zone to operate at higher neutron fluxes than would otherwise be possible. The net result is a flatter neutron flux profile across the reactor in a radial direction.

The overall effect in CANDU reactors is that the average neutron flux is about 60% of the maximum neutron flux. The ratio of $\phi_{average} / \phi_{maximum}$ is thus 0.60. Note that for a cylindrical reactor with no flux flattening the value of $\phi_{average} / \phi_{maximum}$ would be 0.27. This is the inverse of $\phi_{maximum} / \phi_{average}$ which is 3.64. This means that a CANDU reactor can produce just over twice the amount of power as a simple cylindrical reactor of the same size and design but without flux flattening devices. There is thus a great economic benefit derived from neutron flux flattening.

EFFECT OF FUEL RODS

The effect of fuel rods on the neutron flux is to suppress the thermal flux since thermal neutrons are absorbed. The thermal neutron flux profile thus varies between the fuel and the moderator with a series of dips wherever there is a fuel rod and a series of peaks wherever there is moderator. The inverse occurs with the fast neutron flux. Fast neutrons are created in the fuel so that fast neutron flux peaks in the fuel. In the moderator the fast neutrons are thermalised and the fast neutron flux dips while thermal neutron flux builds up.

EFFECT OF CONTROL RODS

The effect of control rods is to suppress the neutron flux since neutrons are strongly absorbed by them. This can have the effect of enhancing the surrounding neutron flux if adjustments are made to the overall reactor power. For this reason it is undesirable to have only a few control rods. Most reactors have a large number of control rods so that the overall effect on the neutron flux is more uniform across the reactor. The PWR in fact has a cluster of control rods which are inserted into each fuel assembly to distribute the effect as smoothly as possible across the reactor.

CHEMICAL SHIM

In water reactors it is convenient to add a neutron absorber to the moderator or coolant to

absorb neutrons uniformly. The reactor may then operate generally with the control rods in their optimum position so as to minimise neutron flux distortions. Soluble neutron absorbers or reactor poisons, such as boric aid, lose their effectiveness as they absorb neutrons. The burnup of the reactor poison is compensated by the burnup of reactor fuel. As the fuel is depleted less soluble poison is required but it itself is becoming depleted so the two effects counterbalance one another. Slight changes in poison concentration are made continuously by the reactor chemical treatment facility hence the term chemical shim when reference is made to reactivity control using soluble neutron absorbers. This method is widely used on PWR's which are refuelled once a year. CANDU reactors which utilise on-line refuelling do not need such treatment except after their first commissioning with a complete charge of new fuel.

ONE GROUP REACTOR EQUATION

ONE GROUP STEADY STATE DIFFUSION EQUATION

$$D\nabla^2\phi - \Sigma_a\phi = -S$$

IN A CRITICAL REACTOR $(k=1)$

$$S = \eta\,\Sigma_{a\ FUEL}\,\phi$$

η = NEUTRONS EMITTED PER NEUTRON ABSORBED

$$S = \eta\,\frac{\Sigma_{a\ FUEL}}{\Sigma_{a\ REACTOR}}\,\Sigma_{a\ REACTOR}\,\phi$$

f = FUEL UTILIZATION $\Sigma_{a\ FUEL}/\Sigma_{a\ REACTOR}$

$$S = \eta f\,\Sigma_{a\ REACTOR}\,\phi$$

IN AN INFINITE REACTOR

TOTAL NEUTRONS ABSORBED $= \Sigma_{a\ REACTOR}\,\phi$

NEUTRONS ABSORBED IN FUEL $= f\,\Sigma_{a\ REACTOR}\,\phi$

NEUTRONS RELEASED BY FISSION $= \eta f\,\Sigma_{a\ REACTOR}\,\phi$

BY DEFINITION OF k_∞

$$k_\infty = \frac{\text{NEUTRONS IN CURRENT GENERATION}}{\text{NEUTRONS IN PREVIOUS GENERATION}}$$

$$= \eta f\,\Sigma_{a\ REACTOR}\,\phi\,/\,\Sigma_{a\ REACTOR}\,\phi$$

$$= \eta f$$

$$S = k_\infty\,\Sigma_{a\ REACTOR}\,\phi$$

ONE GROUP STEADY STATE DIFFUSION EQUATION

$$D\nabla^2\phi - \Sigma_a\phi = -S$$

$$D\nabla^2\phi - \Sigma_{a\ REACTOR}\,\phi = -k_\infty\Sigma_{a\ REACTOR}\,\phi$$

$$D\nabla^2\phi + k_\infty\Sigma_{a\ REACTOR}\,\phi - \Sigma_{a\ REACTOR}\,\phi = 0$$

$$D\nabla^2\phi + (k_\infty - 1)\,\Sigma_{a\ REACTOR}\,\phi = 0$$

$$\nabla^2\phi + (k_\infty - 1)\,\frac{\Sigma_{a\ REACTOR}}{D}\,\phi = 0$$

BUT $\frac{D}{\Sigma_a} = L^2$

$$\nabla^2\phi + (k_\infty - 1)\,\frac{1}{L^2}\,\phi = 0$$

NOW $B^2 = \frac{k_\infty - 1}{L^2}$ BY DEFINITION

HENCE ONE GROUP REACTOR EQUATION IS :

$$\nabla^2\phi + B^2\phi = 0$$

THE SLAB REACTOR

ONE GROUP REACTOR EQUATION

$$\nabla^2\phi + B^2\phi = 0$$

FOR INFINITE BARE SLAB OF THICKNESS a

$$\frac{d^2\phi}{dx^2} + B^2\phi = 0$$

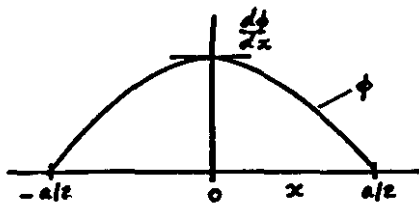BOUNDARY CONDITIONS : ϕ IS ZERO AT SURFACE

ϕ IS MAX. AT CENTRE

$$\phi_{(x=a/2)} = 0$$

$$\frac{d\phi}{dx}_{(x=0)} = 0$$

GENERAL SOLUTION IS :

$$\phi_{(x)} = A\cos Bx + C\sin Bx$$

SOLUTION WITH BOUNDARY CONDITIONS IS :

$$B = \frac{\pi}{2}$$
$$x = \frac{a}{2}$$

$$\phi_{(a/2)} = A\cos\left(\frac{Ba}{2}\right)$$

$$\phi_{(x)} = A\cos\left(\frac{\pi x}{a}\right)$$

INFINITE BARE SLAB EQUATION

$$\frac{d^2\phi}{dx^2} + B^2\phi = 0$$

$$B^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

AND $B^2 = \left(\frac{\pi}{a}\right)^2$

POWER IN REACTOR IS GIVEN BY

$$P = E_{CONVERSION\ FACTOR}\,\Sigma_f\int_{-a/2}^{a/2}\phi_{(x)}\,dx$$

$$= E_R\,\Sigma_f\int_{-a/2}^{a/2}A\cos\frac{\pi x}{a}\,dx$$

$$= E_R\,\Sigma_f\left[A\frac{a}{\pi}\sin\frac{\pi x}{a}\right]_{-a/2}^{a/2}$$

$$= E_R\,\Sigma_f\left[\frac{Aa}{\pi}(1+1)\right]$$

$$= 2a\,E_R\,\Sigma_f\,A\,/\,\pi$$

$$A = \pi P\,/\,2a\,E_R\,\Sigma_f$$

SOLUTION FOR BARE SLAB

$$\phi_{(x)} = A\cos\left(\frac{\pi x}{a}\right)$$

$$\phi_{(x)} = \left(\pi P\,/\,2a\,E_R\,\Sigma_f\right)\cos\left(\frac{\pi x}{a}\right)$$

$$\phi_{(x)} = \left(1.57 P\,/\,a\,E_R\,\Sigma_f\right)\cos\left(\frac{\pi x}{a}\right)$$

OTHER REACTOR SHAPES

THE SLAB REACTOR

$$\phi = \left({}^{1.57P}/_{\alpha E_R \Sigma_f}\right) \cos\left(\frac{\pi x}{\alpha}\right)$$

$$\phi_{MAX}/\phi_{AVE} = 1.57$$

THE SPHERICAL REACTOR

$$\phi = \left({}^{P}/_{4 E_R \Sigma_f R^2}\right)\left(\frac{1}{r}\right) \sin\left(\frac{\pi r}{R}\right)$$

$$\phi_{MAX}/\phi_{AVE} = 3.29$$

THE INFINITE CYLINDRICAL REACTOR

$$\phi = \left({}^{0.738\,P}/_{E_R \Sigma_f R^2}\right) J_o\left(\frac{2.405\,r}{R}\right)$$

$$\phi_{MAX}/\phi_{AVE} = 2.32$$

THE FINITE CYLINDRICAL REACTOR

$$\phi = \left({}^{3.63P}/_{E_R \Sigma_f V}\right) J_o\left(\frac{2.405\,r}{R}\right) \cos\left(\frac{\pi z}{H}\right)$$

$$\phi_{MAX}/\phi_{AVE} = 3.64$$

THE RECTANGULAR REACTOR

$$\phi = \left({}^{3.87P}/_{E_R \Sigma_f V}\right) \cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$$

$$\phi_{MAX}/\phi_{AVE} = 3.88$$

MAXIMUM TO AVERAGE FLUX

BARE SPHERICAL REACTOR

$$\phi_{MAX} = \frac{P}{4 E_R \Sigma_f R^2} \lim_{r \to 0} \frac{\sin(\pi r/R)}{r}$$

$$= \frac{\pi P}{4 E_R \Sigma_f R^2 R}$$

AVERAGE VALUE OF FLUX

$$\phi_{AVE} = \frac{\int \phi\, dV}{V}$$

REACTOR POWER IS GIVEN BY :

$$P = E_R \Sigma_f \int \phi\, dV$$

$$\int \phi\, dV = P / E_R \Sigma_f$$

SUBSTITUTING INTO PREVIOUS EQUATION

$$\phi_{AVE} = \frac{P}{E_R \Sigma_f V}$$

$$\phi_{MAX}/\phi_{AVE} = \frac{\pi P}{4 E_R \Sigma_f R^3} / \frac{P}{E_R \Sigma_f V}$$

$$= \frac{\pi V}{4 R^3}$$

$$= \frac{\pi}{4 R^3}\left(\frac{4 \pi R^3}{3}\right)$$

$$= \frac{\pi^2}{3} = 3.29$$

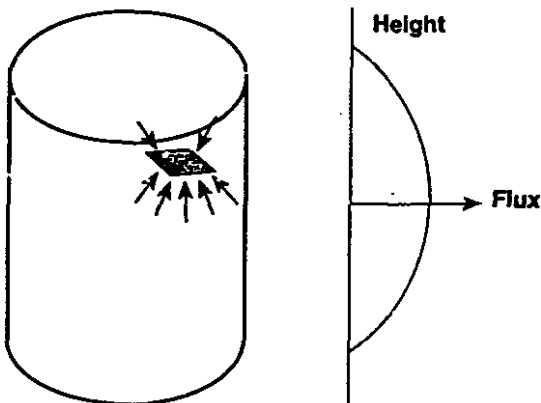### Variation of the Thermal Neutron Flux along the Axis of a Cylindrical Reactor

Height

Flux

Fig. 6.1

Variation of Thermal Neutron Flux in Axial and Radial Directions in a Cylindrical Reactor
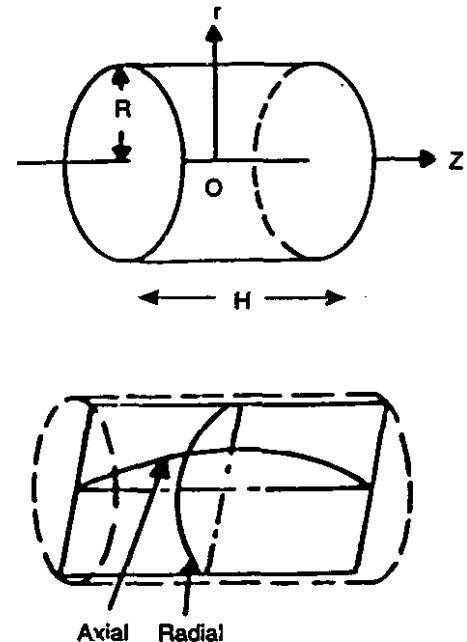

r

R

O

z

H

Axial   Radial

Fig. 6.2

ONE GROUP CRITICAL EQUATION

FOR CRITICALITY (STEADY STATE CONDITIONS)

$$B^2 = \frac{k_\infty - 1}{L^2}$$

B^2 DEPENDS ON REACTOR DIMENSIONS

$(k_\infty - 1)/L^2$ DEPENDS ON MATERIAL PROPERTIES

SPECIFY ONE AND CALCULATE THE OTHER

ONE GROUP CRITICAL EQUATION

$$B^2 L^2 = k_\infty - 1$$
$$k_\infty = B^2 L^2 + 1$$
$$\frac{k_\infty}{1 + B^2 L^2} = 1$$

ONE GROUP REACTOR EQUATION

$$\nabla^2 \phi + B^2 \phi = 0$$

NUMBER OF NEUTRONS ABSORBED

$$= \Sigma_a \int_v \phi \, dV$$

NUMBER OF NEUTRONS LEAKING FROM SYSTEM

$$= \int_A J n \, dA$$
$$= \int_v \text{div} \, J \, dV$$
$$= -D \int_v \nabla^2 \phi \, dV$$
$$= D \int_v B^2 \phi \, dV$$
$$= D B^2 \int_v \phi \, dV$$

PROBABILITY OF ABSORPTION (NON-LEAKAGE)

$$P_L = \frac{\text{NUMBER ABSORBED}}{\text{NUMBER ABSORBED} + \text{NUMBER LEAKING}}$$
$$= \Sigma_a \int_v \phi \, dV / \left[\Sigma_a \int_v \phi \, dV + D B^2 \int_v \phi \, dV \right]$$
$$= \frac{\Sigma_a}{\Sigma_a + D B^2}$$
$$= \frac{1}{1 + B^2 L^2} \qquad \text{SINCE } L^2 = \frac{D}{\Sigma_a}$$

ONE GROUP CRITICAL EQUATION

$$1 = \frac{k_\infty}{1 + B^2 L^2} \qquad \frac{1}{k_\infty} = \frac{1}{1 + B^2 L^2}$$

COMBINING THESE EQUATIONS IN CRITICAL SYSTEM

$$1 = k_\infty P_L$$
$$k = k_\infty P_L$$

THE FOUR FACTOR FORMULA

$$k_\infty = \eta f$$

η = NEUTRONS EMITTED PER NEUTRON ABSORBED

f = THERMAL UTILIZATION $\Sigma_{a \, FUEL} / \Sigma_{a \, REACTOR}$

FAST NEUTRONS WILL CAUSE SOME FISSIONS IN U-238 WHICH WOULD OTHERWISE NOT OCCUR

ϵ = FAST FISSION FACTOR

SOME NEUTRONS MAY BE ABSORBED IN U-238 RESONANCES WHILE SLOWING DOWN

p = PROBABILITY OF NOT BEING ABSORBED

p = RESONANCE ESCAPE PROBABILITY

FOR AN INFINITE REACTOR

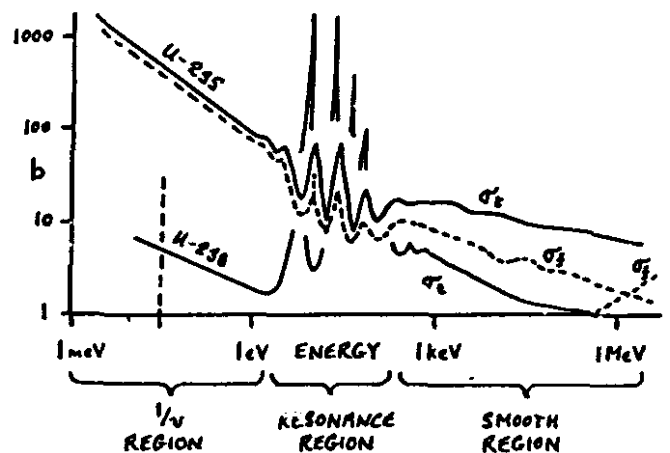$$k_\infty = p \epsilon \eta f \qquad \text{(4 FACTORS)}$$

FOR A REACTOR OF FINITE SIZE

$$k = p \epsilon \eta f \, P_L$$

SIX FACTOR FORMULA

$$k = p \epsilon \eta f \Lambda_f \Lambda_t \qquad \text{(6 FACTORS)}$$

Λ = NON-LEAKAGE PROBABILITY

FISSION CHARACTERISTICS

INTERACTIONS OF IMPORTANCE

σ_s = SCATTERING

$\left. \begin{array}{l} \sigma_\gamma = \text{RADIATIVE CAPTURE} \\ \sigma_f = \text{FISSION} \end{array} \right\} \sigma_a = \text{ABSORPTION}$

CAPTURE / FISSION RATIO : $\alpha = \sigma_\gamma / \sigma_f$
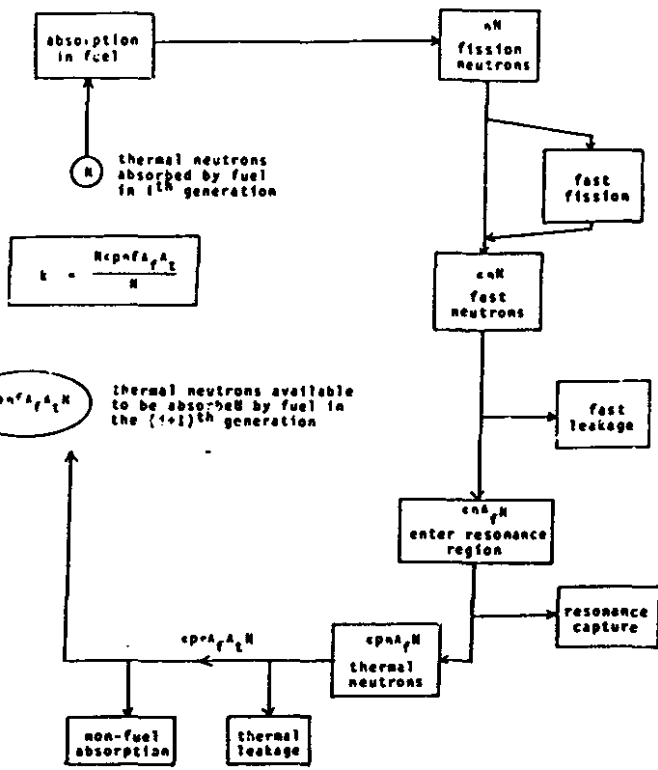
PROBABILITY OF FISSION : $p = \sigma_f / \sigma_a$

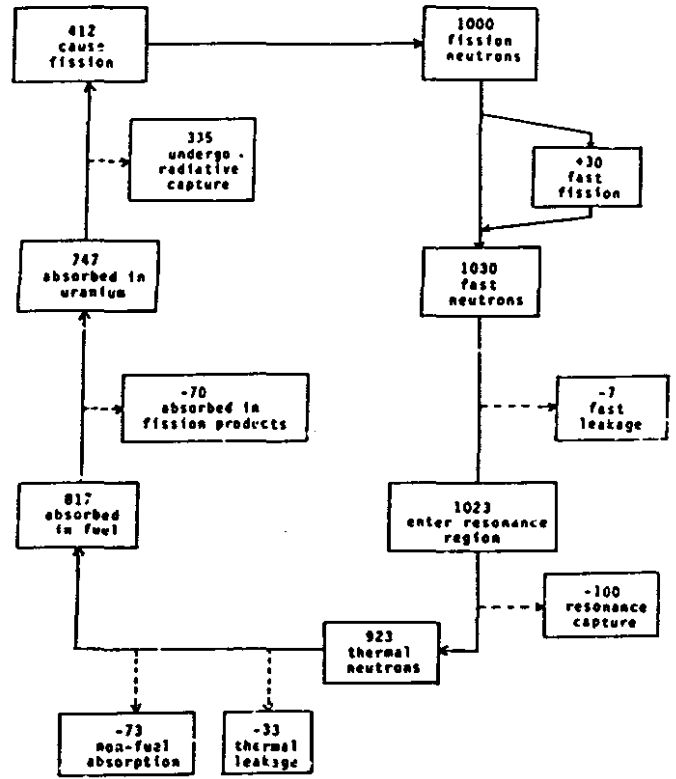**Figure 5.2** Multiplication factor in neutron cycle

Boxes (left diagram):
- absorption in fuel
- N — thermal neutrons absorbed by fuel in the ith generation
- $t = \dfrac{N \epsilon p \eta f \Lambda_f \Lambda_t}{N}$
- nN fission neutrons
- fast fission
- ϵnN fast neutrons
- $\epsilon p \eta f \Lambda_f \Lambda_t N$ — thermal neutrons available to be absorbed by fuel in the $(i+1)^{th}$ generation
- fast leakage
- $\epsilon n \Lambda_f N$ enter resonance region
- resonance capture
- $\epsilon p \eta \Lambda_f N$
- $\epsilon p n \Lambda_f N$ thermal neutrons
- non-fuel absorption
- thermal leakage

Figure 5.1 Neutron life cycle

Boxes (right diagram):
- 412 cause fission
- 1000 fission neutrons
- 335 undergo radiative capture
- +30 fast fission
- 747 absorbed in uranium
- 1030 fast neutrons
- -70 absorbed in fission products
- -7 fast leakage
- 817 absorbed in fuel
- 1023 enter resonance region
- -100 resonance capture
- 923 thermal neutrons
- -73 non-fuel absorption
- -33 thermal leakage

Neutron Multiplication Factor

Neutron multiplication factor

$$k_\infty = \epsilon p \eta f \quad \text{(4 Factor)}$$

$$k = \epsilon p \eta f \Lambda_f \Lambda_t \quad \text{(6 Factor)}$$

ϵ = Fast fission factor

p = Resonance escape probability

η = Reproduction factor

$\quad = \nu \Sigma_f^{FUEL} \Sigma_f^{FUEL}$

ν = Neutrons per fission

f = Thermal utilization factor

$\quad = \Sigma_a^{FUEL} / \Sigma_a^{REACTOR}$

Λ_f = Fast neutron non-leakage probability

Λ_t = Slow neutron non-leakage probability

For reactor of finite size

$$k = k_\infty \Lambda_f \Lambda_t$$

k_∞ = k value for infinitely large reactor

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Fission Characteristics

Graph: cross-section b (1, 10, 100, 1000) vs Energy (1 meV, 1 eV, Energy, 1 keV, 1 MeV), showing U-235 and U-238 curves with σ_t, σ_f labels.

Regions: 1/v region, Resonance region, Smooth region

Interactions of importance

σ_s = Scattering

σ_γ = Radiative capture

σ_f = Fission

$\left. \right\}$ σ_a = Absorption

Capture/fission ratio: $\alpha = \sigma_\gamma / \sigma_f$

Probability of fission: $p = \sigma_f / \sigma_a$

Surface-Volume Ratio

Cube of side D volume 100
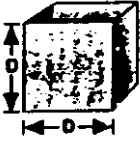
$$Surface = 6D^2$$
$$Volume = D^3$$
$$D^3 = 100 \quad \therefore D = 4.64$$
$$S = 6(4.64)^2 = 129$$
$$S:V \ Ratio = 129/100 = 1.29$$

Cylinder of length D, diameter D, volume 100

$$Surface = 2\frac{\pi}{4}D^2 + \pi D(D) = \frac{3}{2}D^2$$
$$Volume = \frac{\pi}{4}D^2 D = \frac{\pi}{4}D^3$$
$$D^3 = 100(\frac{\pi}{4}) \quad \therefore D = 5.03$$
$$S = \frac{3}{2}\pi(5.03)^2 = 119$$
$$S:V \ Ratio = 119/100 = 1.19$$

Sphere of diameter D, volume 100

$$Surface = \pi D^2$$
$$Volume = \frac{\pi}{6}D^3$$
$$D^3 = 100(\frac{6}{\pi}) \quad \therefore D = 5.76$$
$$S = \pi(5.7C)^2 = 104$$
$$S:V \ Ratio = 104/100 = 1.04$$

Sphere

Volume: $\frac{\pi}{6}D^2$

Surface: πD^2

Surface/ volume ratio: $\pi D^2 / \frac{\pi}{6}D^2$

$$= 6/D$$

if D = 1		Ratio =	6
if D = 2		Ratio =	3
if D = 3		Ratio =	2
if D = 4		Ratio =	1.5
if D = 5		Ratio =	1.2
if D = 6		Ratio =	1

Path of a Neutron from Birth to Absorption

A   A fast neutron is born from fission

The neutron moves through the moderator and loses energy in each collision with a moderator atom

B   The neutron reaches thermal energy

The neutron diffuses through the moderator

C   The Neutron causes fission
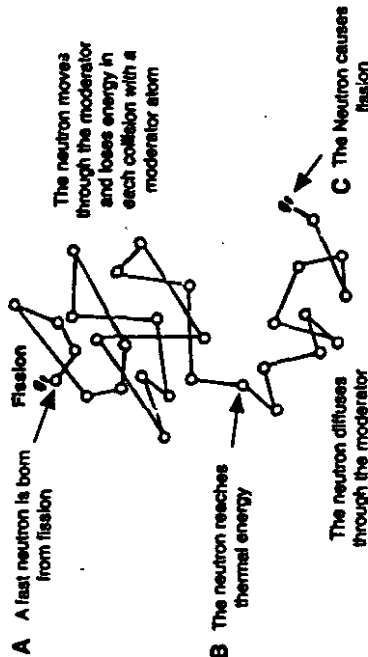
Comparison of Neutron Leakage for Bare and Reflected Cores

Reactor Core

(a)

(b)

Fig. 6.4

256   Nuclear Reactor Theory
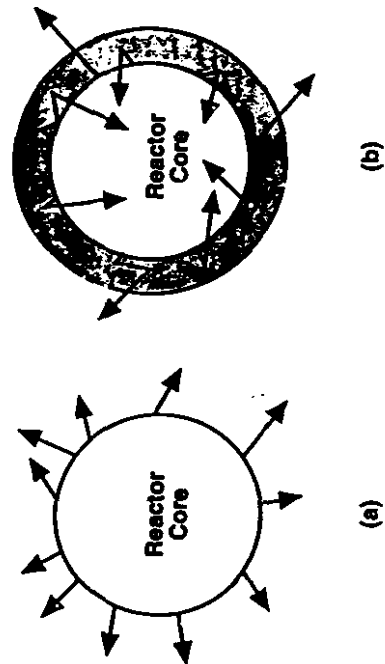
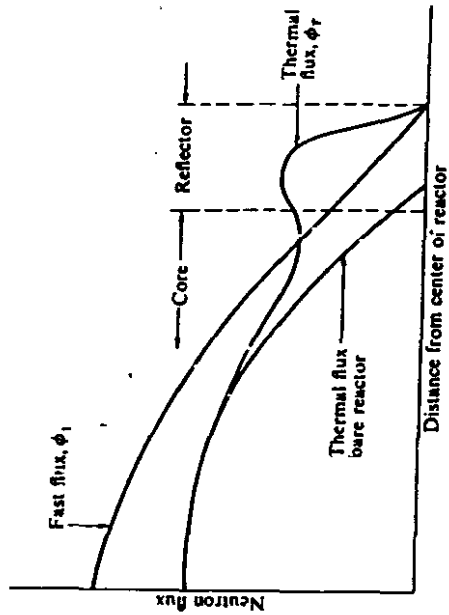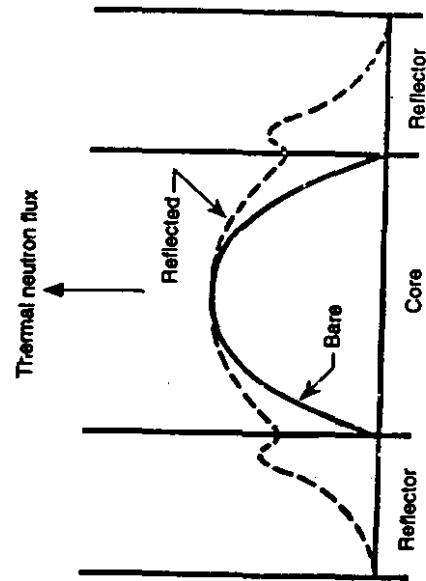

Fig. 6.5 Fast and thermal fluxes in a reflected thermal reactor and the thermal flux in the equivalent reactor.

Flux Distribution in Reflected Reactor
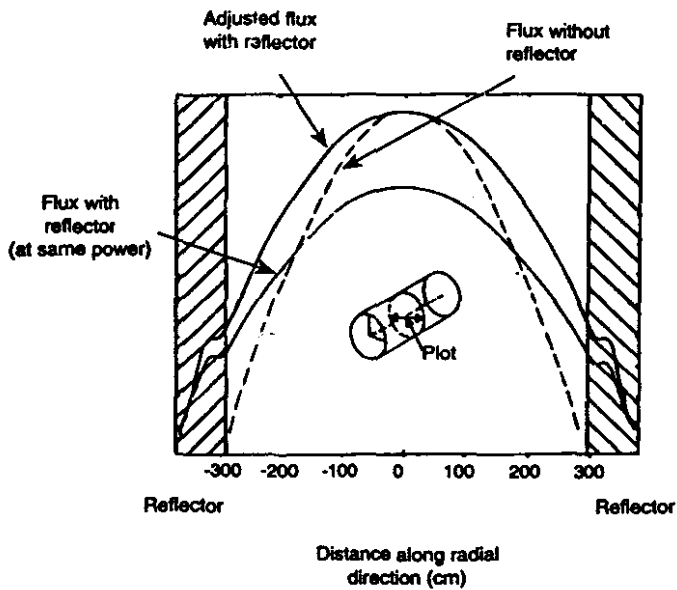

Effect of Reflector on Shape of Radial Flux

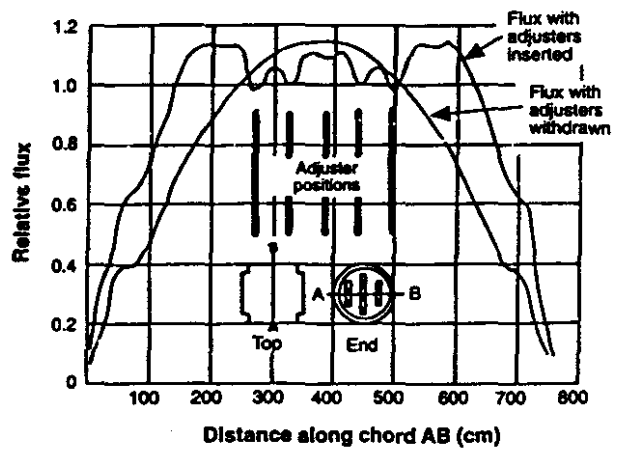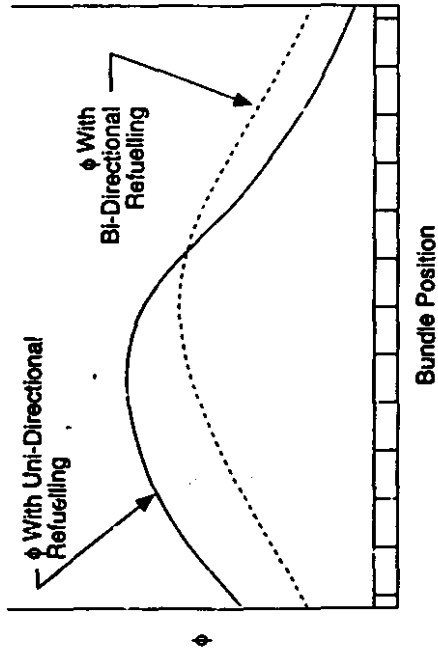
Fig. 6.5

Flux Flattening Produced by Adjuster Rods

Fig. 6.6

Asymmetry Produced by Uni-Directional Refuelling

φ With Bi-Directional Refuelling

φ With Uni-Directional Refuelling

Bundle Position

Effect of Bi-Directional Refuelling in Flattening Axial Flux Shape

Theoretical flux shape with uniform fuelling

Total flux

Flux from channels fuelled right to left

Flux from channels fuelled left to right

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Bundle positions along channels
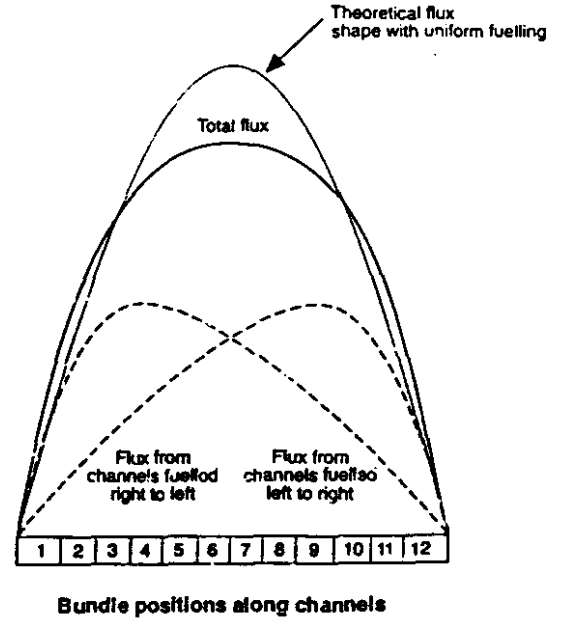
Fig. 6.7

Flux Flattening Produced by Differential Fuelling

Inner zone

Reactor face

Outer zone

Flux with differential fuelling

φ

Flux without differential fuelling

Inner zone

Radial position

Fig. 6.9

Flux Flattening

Reflector

Adjuster rods

Radial

Radial and axial

Bi-directional fuelling

Differential burn-up

Axial only

Radial only

- - - - - - - - Without flattening
—·—·—·—·— With flattening
——————— With flattening and power increase
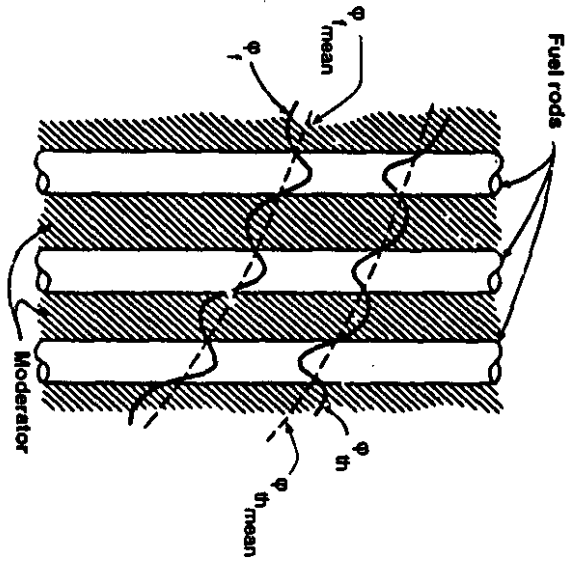
OH 6.2

Flux Variation in Fuel
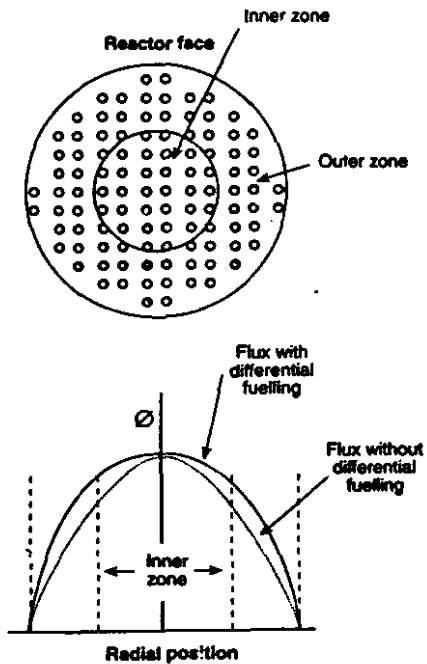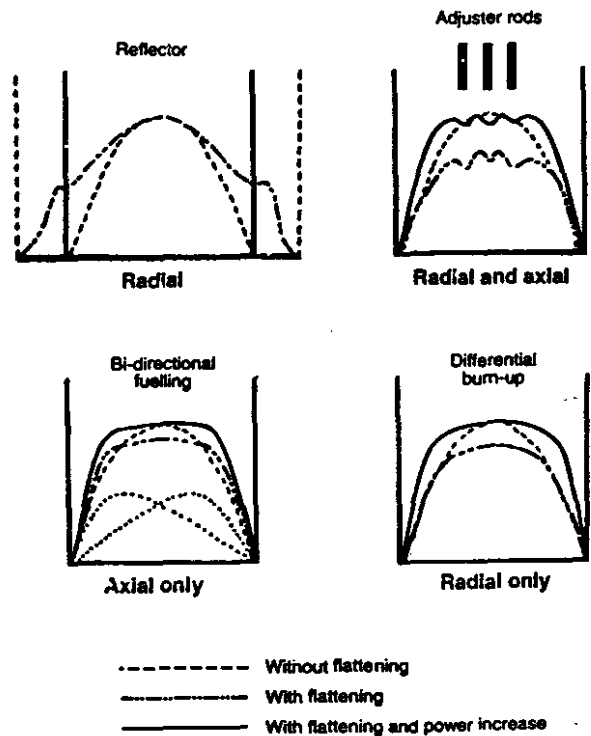
Flux Flattening in CANDU Reactors

	Reflector	Bi-directional fuelling	Adjusters	Differential burnup	$\frac{\phi_{avg}}{\phi_{max}}$
NPD	axial & radial	x			42%
Gentilly	radial			x	50%
Pickering - A	radial	x	x		57%
Pickering-B	radial		x		~60%
Bruce - A	radial	x		x	~59%
Bruce-B	radial		x		~60%
Darlington	radial	x	x	x	~60%
Point Lepreau	radial		x	x	~60%

Depression of the Thermal Neutron Flux in the Interior of Fuel Bundle

Neutron Flux in Core