Module 1

Nuclear Structure

July, 1997 (R-1)
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Notes & References

1.1 MODULE OVERVIEW

The purpose of this module is to provide basic information about the structure of the atomic nucleus and the principles governing the release of energy in nuclear processes such as radioactivity and fission. Some of this information, such as the description of the three common types of radioactive emissions (alpha, beta, gamma) and the law governing the rate of radioactive decay, is directly connected to operational issues such as radiological safety and xenon transients. Other information, such as mass-energy equivalence and the concept of nuclear binding energy, is not so obviously relevant to the day-to-day operation of the reactor, but serves as the foundation for topics covered in later modules. This includes the assessment of the energy obtainable from a given mass of fuel and the reason why a chain reaction can take place with uranium fuel.

1.2 MODULE OBJECTIVES

After studying this module, you should be able to:

i) Define and use the $^{A}_{Z}X$ notation.

ii) Use the atomic mass unit.

iii) Define alpha, beta and gamma emissions.

iv) State the basic law governing radioactive decay.

v) Define the decay constant (λ) and half-life ($t_{1/2}$) of a radioisotope.

vi) Calculate the activity of a radioisotope of a given half-life.

vii) State the mass-energy equivalence principle.

viii) Define mass defect and binding energy.

ix) Explain the origin of energy releases in nuclear reactions.

x) Describe how the binding energy per nucleon varies with atomic mass number.

xi) Explain how the stability of nuclei varies in terms of their neutron-proton ratio.

1.3 THE ATOMIC NUCLEUS AND ITS CONSTITUENTS

Atoms

Nucleus

The matter around us is made up of minute entities called ***atoms***, about 10^{-10} meters in diameter. Nearly all the mass of an atom is concentrated in a very small, positively charged, ***nucleus*** at its centre. The diameter of the nucleus is typically in the order of 5×10^{-15} meters. The nucleus is surrounded by a number of very light, negatively charged particles called electrons, which may be imagined as orbiting the nucleus and held in place by electrostatic attraction. The atom as a whole is normally electrically neutral, since the positive charge on the nucleus balances the sum of the electrons' charges.

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One chemical *element* is distinguished from another by the number of electrons in its atom (or, alternatively, by the magnitude of the charge on its nucleus). For example, the simplest element, hydrogen, has only one electron, while uranium has 92. The nucleus is made up of two types of particles which are approximately equal in mass—***protons*** and ***neutrons***.

Atomic number

Atomic mass number

The proton is positively charged. Its charge is as large as that of the electron but of the opposite sign. The neutron is uncharged. The number of protons in a given nucleus is known as the ***atomic number,*** and is represented by the symbol Z. The number of neutrons in a nucleus is known as the ***neutron number***, and is represented by the symbol N. The total number of ***nucleons*** (neutrons plus protons) in the nucleus is called the ***atomic mass number, A.*** Hence,

$$A = N + Z$$

Isotopes

Radioactive decay

An atom with a given Z and A value is called a ***nuclide***. Nuclides with the same number of protons (that is, the same Z value) but with different numbers of neutrons (that is, different A values) are known as ***isotopes***. There are, for example, three possible isotopes of the element hydrogen, as illustrated in Figure 1.1. (The third isotope, tritium, does not occur naturally, but is produced in CANDU reactors when neutrons are captured by the deuterium isotope). Isotopes may be stable (not subject to ***radioactive decay)*** or unstable (subject to *radioactive decay).* Tritium, for example, is radioactive, while the other two isotopes of hydrogen are stable. The maximum number of isotopes of an element ranges from 3 for hydrogen to 26 for tin, with an average of about 10 isotopes per element. The standard way to symbolize an isotope is:

$${}^{A}_{Z}X$$

where X is the chemical symbol for the element (for a listing of chemical symbols, see Appendix A). A nuclide is often symbolized by listing only its chemical symbol and its atomic mass number, for example, U-235, since the chemical symbol implies the atomic number.

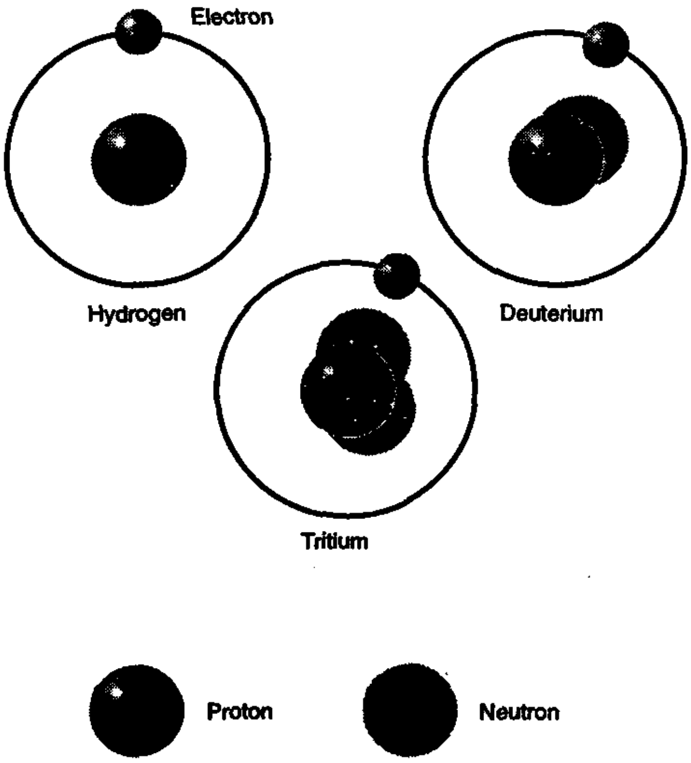

Figure 1.1: The isotopes of hydrogen

The masses and charges for the three constituents of the atom are listed in the table below. Note that the mass of the neutron is marginally (about 0.14%) greater than that of the proton.

Mass and charge table

Table 1.1
Masses and charges of the atomic constituents

	Mass (kg)	Charge (coulombs)
Proton	1.67265×10^{-27}	1.602×10^{-19} (positive)
Neutron	1.67495×10^{-27}	0
Electron	9.10953×10^{-31}	1.602×10^{-19} (negative)

1.4 NUCLEAR MASS SCALE

Atomic mass unit

Since atomic masses are so tiny, it is convenient to introduce a very small unit, known as the ***atomic mass unit*** (u), to simplify the arithmetic. This unit is defined by stipulating the mass of the ***neutral atom*** of the isotope carbon-12 to be precisely 12 u. The equivalence between the atomic mass unit and the kilogram can be shown to be:

$$1\ u \;=\; 1.660540 \times 10^{-27}\ kg$$

The masses of the atomic constituents in atomic mass units are:

Proton	1.0072765 u
Neutron	1.0086650 u
Electron	0.0005486 u

The masses of stable isotopes are provided in atomic mass units in the General Electric "Chart of the Nuclides", issued as supplementary material to this course.

1.5 RADIOACTIVITY

All naturally occurring nuclides heavier than lead (Z=82), along with a few lighter nuclides, are ***unstable***. As a result a nucleus of such a nuclide can spontaneously change itself ("decay") into the nucleus of another isotope by emitting either an ***alpha particle*** (actually a helium nucleus) or a ***beta particle*** (a fast electron). Nuclides which undergo radioactive decay are known as ***radionuclides*** or ***radioisotopes***.

Examples of the two types of decay are shown below:

Alpha decay

$$\textit{Alpha decay:}\ {}^{235}_{92}U \rightarrow {}^{231}_{90}Th + {}^{4}_{2}\alpha$$

The symbol α represents the alpha particle which is a composite unit containing 2 protons and 2 neutrons. The emission of an alpha particle from the uranium nucleus leaves a nucleus of Th-231.

Beta decay

$$\textit{Beta decay:}\ {}^{116}_{49}In \rightarrow {}^{116}_{50}Sn + {}^{0}_{-1}\beta$$

Symbol β represents the beta particle which, as mentioned, is simply an ordinary electron with a mass of approximately zero and a negative charge of one unit. When beta decay occurs, a neutron in the nucleus changes itself into a proton plus an electron, as shown below

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta$$

Neutrino

In reality, the process is rather more complicated for it turns out that another particle, called a ***neutrino*** (strictly speaking, an ***antineutrino***) is always emitted in addition to the beta particle. The beta decay above should therefore formally be written as

$${}^{116}_{49}In \rightarrow {}^{116}_{50}Sn + {}^{0}_{-1}\beta + {}^{0}_{0}\nu$$

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where the symbol ${}^{0}_{0}\nu$ stands for the antineutrino, which has a charge of zero and (probably) zero mass.

Gamma ray

In most cases of alpha or beta decay, the new nucleus created (known as the ***daughter*** nucleus) is left with excess energy; it is said to be in an excited state. It gets rid of this excess energy ("decays to its ground state") by emitting ***a gamma ray***, which is a photon of radiation. Gamma rays, which are represented by the symbol γ, are similar to x-rays, but have a higher energy. Usually (though not always) gamma rays are emitted instantaneously, that is, within 10^{-14} seconds of the formation of the daughter nucleus. The beta decay of a typical "parent" nucleus, followed by the emission of a gamma ray, is illustrated in Figure 1.2. As gamma rays are highly penetrating, they constitute an important part of the radiation hazard associated with fuel which has been irradiated for long periods in the reactor.

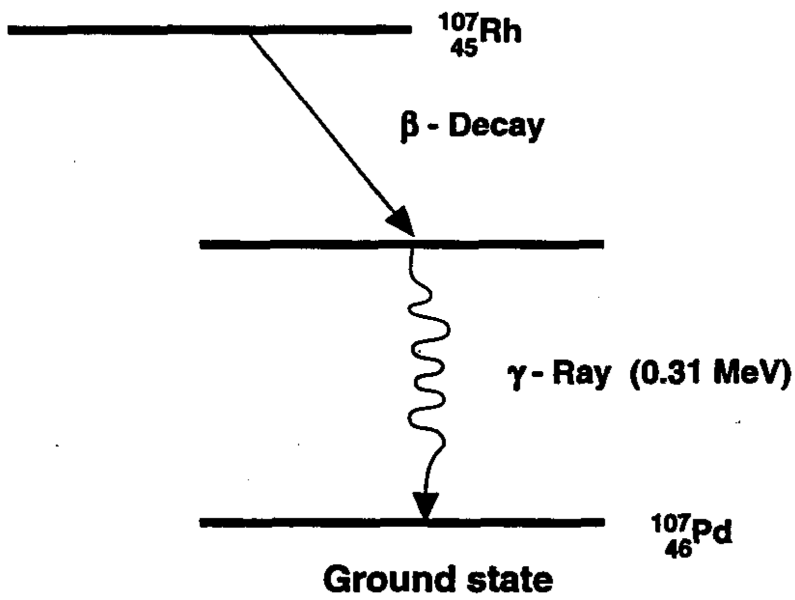

Figure 1.2 : Beta decay followed by gamma ray emission

Notes & References

Radioactivity is governed by only one fundamental law: the probability of a radionuclide decaying per unit time is constant and independent of external conditions. This constant is called the ***decay constant*** and is symbolized by λ. If we take a sample consisting of N atoms of a radionuclide of decay constant λ, the rate of decay of the sample (number of atoms decaying per second) is equal to λN. Since N is decreasing, the equation for the rate of change of N is

Decay constant

$$\frac{dN}{dt} = -\lambda N \qquad (1.1)$$

This equation can be solved to give N(t), the number of radioactive atoms still remaining in the sample after some time t has elapsed. Assuming that the number of atoms at time t = 0 was N_0, the solution is

$$N(t) = N_o e^{-\lambda t} \qquad (1.2)$$

Thus, the strength of the source decays exponentially with time. The decay rate of a source is often specifi.e.d in terms of a parameter known as the ***half-life*** ($t_{1/2}$), which is the ***time required for the number of radioactive atoms to decay to half its initial value***. We can derive a simple relationship between half-life and decay constant by noting that when t in equation (1.2) above is equal to the half-life, the number of radioactive atoms N(t) will be equal to $N_o/2$.

Half-life

Substituting these values in the equation, we have

$$N_o / 2 = N_o e^{-\lambda t_{1/2}}$$

or

$$e^{-\lambda t_{1/2}} = 1 / 2$$

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Inverting,

$$e^{\lambda t_{1/2}} = 2$$

Taking natural logarithms of both sides

$$\lambda t_{1/2} = \ln 2$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad (1.3)$$

If we want to know what fraction of the active atoms is still present after a given number of half-lives, we can substitute $\lambda = 0.693/t_{1/2}$ in equation (1.2) to obtain

$$N(t) = N_o e^{-0.693t/t_{1/2}} \qquad (1.4)$$

For example, after 4 half-lives, that is, at $t = 4\ t_{1/2}$, the fraction of the active atoms still present is

$$\frac{N(t)}{N_o} = e^{-0.693t/t_{1/2}} = e^{-(0.693 \times 4)} = 0.0625$$

Alternatively, since the source strength will decrease by a factor of $^1/_2$ in each half-life, the total decrease factor over 4 half-lives is $(^1/_2)^4 = {}^1/_{16} = 0.0625$ (see Figure 1.3).

Activity

The magnitude of the quantity dN/dt, the number of decays taking place per second, is known as the ***activity*** of the source. Historically, activity was specified in terms of a unit called the curie (Ci), which was defined as a decay rate of 3.7×10^{10} disintegrations per second. This unit has been superseded by the ***becquerel (Bq)***, which is simply defined as

Becquerel

1 becquerel = 1 disintegration per second

According to equation (1.1), since the magnitude of the activity equals the product λN, we can calculate the activity of a radioactive sample by using the relation

$$Activity = \lambda N \qquad (1.5)$$

where N is the number of active atoms present when the activity is being measured.

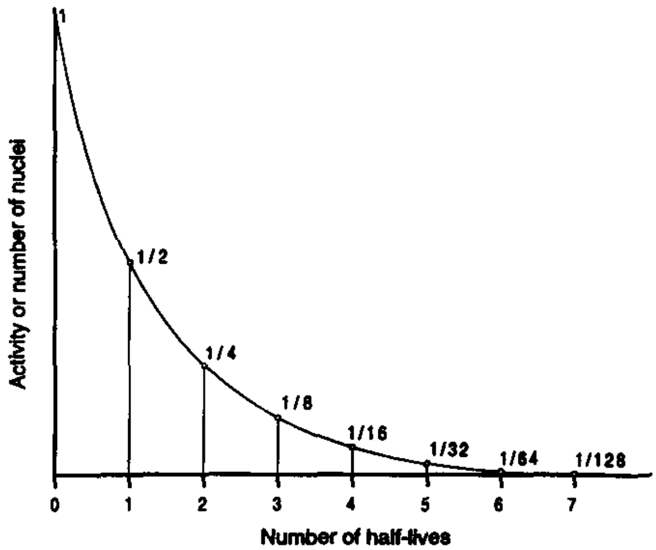

Figure 1.3: Representation of exponential radioactive decay in terms of half-lives

Notes & References

1.6 EQUIVALENCE OF MASS AND ENERGY

In order to understand the source of the energy which is released in radioactive decay (or in nuclear fission), we must consider the principle of the equivalence of mass and energy, discovered by Einstein.

Quantitatively, if a certain amount of mass Δm is "lost" when a reaction (such as a radioactive decay) takes place, the amount of energy, ΔE, liberated as a result of this mass being converted into energy is:

Mass - energy equivalence

$$\Delta E = \Delta m \times c^2 \qquad (1.6)$$

where c is the speed of light (3×10^8 meters per second). ***If the mass is quoted in kilograms, the energy will be provided in joules.***

Electron volt

Since the amounts of energy associated with reactions on the atomic or nuclear scale tend to be rather small, it is customary to measure them by a special unit called the ***electron volt*** (abbreviated ***eV***). This is the amount of kinetic energy gained by an electron (or a proton) when it is allowed to accelerate through a potential difference of 1 volt. The equivalence between eV and joules is:

$$\mathbf{1\ eV = 1.6022 \times 10^{-19}\ joules}$$

Commonly used multipliers of the electron volt are:

$$\mathbf{1\ kilo\text{-}electron\ volt\ (keV) = 10^3\ eV}$$

$$\mathbf{1\ mega\text{-}electron\ volt\ (MeV) = 10^6\ eV}$$

The number of MeV corresponding to 1 atomic mass unit is:

$$\mathbf{1\ u = 931.5\ MeV}$$

1.7 MASS DEFECT AND BINDING ENERGY

Let us think for a moment about what holds nucleons together in the nucleus. It is known that electrical charges of the same sign ***repel*** one another due to a force known as the ***Coulomb*** force. For the nucleus to stay together, therefore, some other force must act between the nucleons, and it must be strong enough to overcome the mutual repulsion of the protons. This force is known as the ***nuclear*** force. It is a very ***short-range*** force, which essentially acts only between nucleons that are adjacent to one another. Any pair of adjacent nucleons, whether p-p, n-n or p-n, will experience the attractive nuclear force, which is approximately the same for each pairing. Whenever nucleons are added to or taken away from a nucleus, work is done by or against the nuclear force. This work is the origin of the ***binding energy*** of a nucleus, which we will discuss in this section.

Coulomb force

Nuclear force

The mass of a stable nucleus is always **less than the sum of the masses of its individual nucleons**, regarded as independent particles. The **mass defect** is defined as

$$\Delta M = Zm_p + Nm_n - {}^{A}_{Z}M \qquad (1.7)$$

Mass defect

where ${}^{A}_{Z}M$ is the mass of the nucleus whose atomic mass number is A and atomic number Z. N = A-Z is the neutron number and m_p and m_n are the respective masses of an individual proton and neutron.

The ***binding energy (B.E.)*** is the energy equivalent of the mass defect, i.e., the total binding energy of the nucleus is:

$$B.E. = (Zm_p + Nm_n - {}^{A}_{Z}M) \times c^2 \qquad (1.8)$$

Binding energy

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If the masses are expressed in kg, with $c = 3 \times 10^8$ m/s, the energy will be in joules. If the masses are expressed in atomic mass units, the binding energy in MeV is given by

$$B.E. = 931.5(Zm_p + Nm_n - {}^A_Z M) \tag{1.9}$$

An important quantity is the binding energy per nucleon, which is obtained by dividing the binding energy of the nucleus by the number of nucleons, that is,

Binding energy per nucleon

$$B.E./nucleon = B.E.\ of\ nucleus/A \tag{1.10}$$

Energy will be liberated in a nuclear reaction if the total binding energy of the products is greater than the total binding energy of the reactants (that is, if the particles in the nucleus are more strongly bound after the reaction than they were before). As an example, consider what takes place when a neutron and a proton combine to form a nucleus of deuterium:

$$^1_1p + {}^1_0n \rightarrow {}^2_1D$$

The total mass of the reactants is:

$$m_p + m_n = 1.007276 + 1.008665 = 2.015941\ u$$

The mass of the deuterium nucleus is 2.013553 u. Hence,

$$\Delta m = 0.002388\ u$$

and

$$B.E.\ of\ nucleus = 931.5 \times 0.002388$$

$$= 2.224\ MeV$$

Hence, when the neutron and proton combine, the nuclear force pulling them together does 2.224 MeV of work. This work is converted into the ***internal excitation energy*** of the product nucleus. The deuterium nucleus will get rid of its internal excitation energy by emitting a gamma ray.

Fission energy

In the case of ***thermal fission of uranium-235***, the capture of a neutron produces an excitation energy equal to the binding energy of a neutron in the uranium-236 product nucleus (about 6.5 MeV). This is usually enough to cause the nucleus to break into two parts since, as discussed at the end of this section, the large number of protons in this nucleus makes it relatively unstable. As the binding energy ***per nucleon*** in the ***fission product nuclei*** is appreciably greater (by about 0.9 MeV) than the binding energy per nucleon in uranium, the total energy of the products is about $(235 \times 0.9) \sim 200$ MeV greater than that of the reactants. Most of this energy difference is liberated in the form of the ***kinetic energy of the recoiling fission fragments***. (In about 15% of the captures, the uranium-236 nucleus does not undergo fission, and in that case the 6.5 MeV excitation energy is liberated in the form of gamma radiation. This is an example of the radiative capture reaction described in Section 2.7.)

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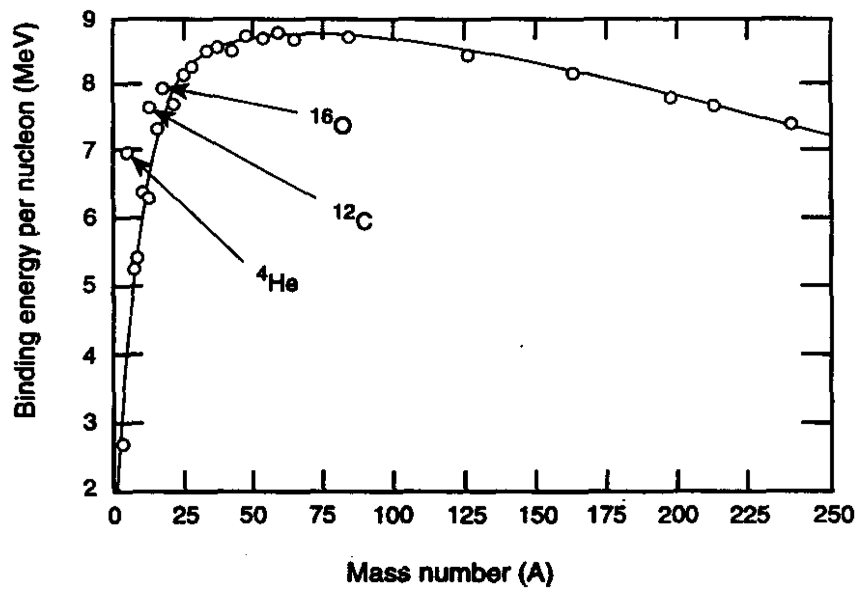

Figure 1.4: Variation of binding energy per nucleon with atomic mass number

The way in which the binding energy per nucleon varies for the elements is shown in Figure 1.4. For the light elements, the binding energy per nucleon increases rapidly with atomic mass number, reaching a maximum of about 8.8 MeV for the elements near iron (A = 56). Thereafter, due to the increasing Coulomb repulsion between the protons, the value decreases slowly, to about 7.6 MeV for uranium. The destabilization caused by the Coulomb force is the reason why so many heavy elements decay by alpha emission. As mentioned previously, the fact that the binding energy per nucleon is greater for medium-mass nuclides than for uranium is the reason why a net liberation of energy occurs when fission takes place.

1.8 NUCLEAR STABILITY

Figure 1.5 shows that stable nuclei with approximately equal numbers of neutrons and protons exist only in the low mass region (e.g., $^{3}_{2}He$, $^{12}_{6}C$, $^{14}_{7}N$, $^{23}_{11}Na$, $^{40}_{20}Ca$). As the number of protons in the nucleus increases, the long range Coulomb forces build up more rapidly than the nuclear forces which only act over a short range. Therefore, for heavier nuclei to remain intact, more neutrons are required to supply binding forces between all particles to overcome the disruptive Coulomb forces. As a result, the N/Z ratio required for stability gradually increases from one in light nuclei to about one and a half in heavier nuclei. This increase in the N/Z ratio for stable nuclei is illustrated in Figure 1.5.

Increase in N/Z ratio

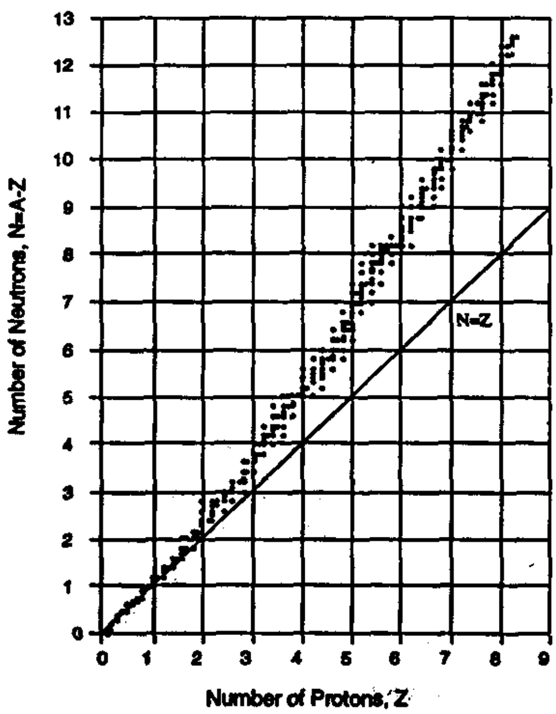

Figure 1.5: N/Z ratios for the stable nuclides

1.9 NUCLEAR ENERGY LEVELS

Excited state

It was mentioned in Section 1.5 that, following an alpha or beta decay, the daughter nucleus is usually left in an ***excited state,*** that is, with a residual energy which it gets rid of by emitting a gamma ray. If we visualize the nucleus as a collection of moving particles bound together by the strong nuclear forces between them, it is clear that the nucleus must have a certain de.g.ree of internal energy. It turns out that this internal energy cannot take just any value, but is restricted to certain particular values characteristic of the nucleus involved. Normally, the nucleus will be in the lowest possible energy level, which is known as its ***ground state.*** If energy is added to the nucleus from outside, for example by bombarding it with high-speed particles, it may jump from its ground state into one of its permitted higher energy levels. These are the ***excited states*** mentioned earlier. The energy of the nucleus in an excited state relative to the ground level is known as the ***excitation energy.*** Normally the nucleus loses its excitation energy almost immediately by emitting a gamma ray. Because the exact values of the permitted energy levels depend on the nucleus involved, the gamma rays emitted by a particular nucleus have energies which are characteristic of that nucleus and no other.

ASSIGNMENT

1. Explain briefly the meaning of each of the following terms:

    a) Atomic number of a nucleus

    b) Atomic mass number of a nucleus

    c) Isotopes of an element

    d) Atomic mass unit (u)

2. a) The isotope Pu-241 is an alpha emitter. Describe what happens in this process and identify the daughter nucleus, using the nuclide chart.

    b) The isotope Mn-57 is a beta emitter. Describe what happens in this process and identify the daughter nucleus.

3. a) Sketch a graph of activity versus time (in half-lives) for a radioactive nuclide, assuming that its activity = A_0 at t = 0.10

    b) Write the relation between the ***activity*** of a sample of radioactive material and the number of atoms in the sample.

    c) Write the expression for the number of active atoms still remaining in the sample at time t, assuming that there were N_0 present at time zero.

d) The isotope xenon-135 has a half-life of 9.6 hours. What is its decay constant?

e) A sample of a radioisotope of half-life 4.5 minutes has 10^6 atoms at t = 0. What is its activity at t = 18 minutes?

4. a) Define the terms ***mass defect*** and ***binding energy*** of a nucleus.

b) Calculate the mass defect and the total binding energy for the carbon-13 nucleus (the mass of a neutral atom of C-13 can be obtained from the Chart of the Nuclides).

c) Calculate the energy released (in MeV) in the following reaction, if the ***nuclear*** masses (in u) are as shown:

$$ {}^{2}_{1}H \quad + \quad {}^{3}_{1}H \quad \rightarrow \quad {}^{4}_{2}He \quad + \quad {}^{1}_{0}n $$

$$ 2.0136 \qquad 3.0155 \qquad 4.0015 \qquad 1.0087 $$

5. Using Einstein's mass-energy equivalence formula, show that 1 u of mass is equivalent to 931.5 MeV. (For this exercise, use a more precise value of c, i.e., 2.997925×10^8 m/s).

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6. State whether each of the following statements is ***true*** or *false*:

   a) The magnitude of the nuclear force between two neutrons is approximately the same as the magnitude of the nuclear force etween two protons.

   b) Energy will be liberated in a nuclear reaction if the total binding energy of the products is less than the total binding energy of the reactants.

   c) The binding energy of a nucleus in the medium-mass range (A = 60 to 100) is about 1 MeV greater than the binding energy of a nucleus such as uranium.

   d) The nuclear force between two nucleons varies as the inverse square of the distance between them.