Flux and Power Mapping in RFSP

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March, 2000

Flux and Power Mapping in RFSP

- Fuel Management Program RFSP has FLUX MAPPING and POWER MAPPING capability
- Alternative to unusual RFSP method of solving the finite-difference diffusion equation in 3 dimensions
- Mapping used to monitor core power distribution under nominal equilibrium core conditions at CANDU 6 sites (Gentilly 2 and Point Lepreau)
- Advantage over diffusion-type calculation is direct inclusion of in-core data in the power calculation

Calculations of the Harmonics (*MONIC)

Steady State Diffusion Equation: $(R-P)\phi=0$

where R is the Removal matrix $\begin{bmatrix} \nabla.D_1\nabla-(\Sigma_{a,1}+\Sigma m) & \nabla.D_2\nabla-\Sigma_{a,2} \\ \Sigma_m & 0 \end{bmatrix}$

P is the Production matrix $\begin{bmatrix} 0 & \nu\Sigma_{f,2} \\ 0 & 0 \end{bmatrix}$

is the Flux vector $\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$

Objective: To find eigenfunctions of $R=\frac{1}{\lambda_i}P\phi_i$

At nth iteration $\phi^n=R^{-1}P\phi^{n-1}=R^{-1}P\Sigma_i A_i\phi_i=\Sigma_i\lambda_i A_i\phi_i$

The eigenfunction with the largest λ_i will emerge and dominates. Solution converges to this predominate mode.

Calculations of the Harmonics (con't)

The Adjoint Flux Vector $\phi^* = (\phi_1^* \; \phi_2^*)$ **satisfy the** $\phi^*(R-P)=0$

Bi-Orthogonality of the natural modes:

$$\int (\phi_{1M}^*(\vec{r})\phi_{2M}^*(\vec{r}))\, P \begin{bmatrix} \phi_{1N}(\vec{r}) \\ \phi_{2N}(\vec{r}) \end{bmatrix} d\vec{r} = 0$$

for any two different harmonics M .NE. N

For a pseudo-one-group flux $\phi_T = \phi_1 + \phi_2$ **, it is self-adjoint:**

$$\int \phi_{TM}(r)\, \nu\Sigma_f(r)\, \phi_{TN}(r)\, dr = 0 \qquad M \neq N$$

Calculations of the Harmonics (con't)

Calculation of the Nth harmonic mode involves subtracting off from the unconverged flux the components of the previous harmonics:

$$\phi'_{uc} = \phi_{uc} - \sum_{I=1}^{N-1} A_I \phi_I$$

The Amplitude A_I of the component of the Ith harmonic determined using the approximate orthogonal property of the total flux:

$$A_I = \frac{\int \phi_{TI}(\vec{r}) \nu \Sigma_f(\vec{r}) \phi_{Tuc}(\vec{dr})}{\int \phi_{TI}(\vec{r}) \nu \Sigma_f(\vec{r}) \phi_{TI}(\vec{r}) \vec{dr}} \qquad I = 1, \ldots, N-1$$

Calculations of the Harmonics (con't)

- **A repetitive "Iterate - Subtraction" procedure forces convergence to the next higher harmonic. Harmonics generated are orthogonal (Gram-Scmidt Orthogonalization Procedure).**

Selection of Mode Set

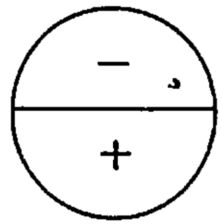
During normal full-power operation the set of modes consists of a fundamental mode (based on a recent core diffusion calculation) and the first 10-14 harmonic modes.

For the normal simulation e.g., a derating during which adjuster banks are withdrawn, a set of 22 flux modes is used:

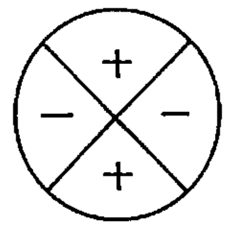
a. fundamental based on diffusion calculation of core state before derating

b. 14 harmonic modes

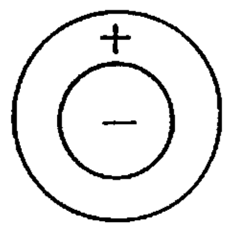
c. 7 power recovery modes with 1 through 7 adjuster banks removed from core



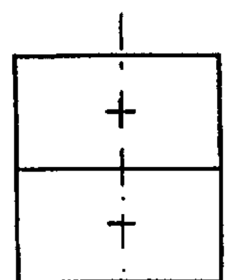
FIRST AZIMUTHAL



SECOND AZIMUTHAL



FIRST RADIAL

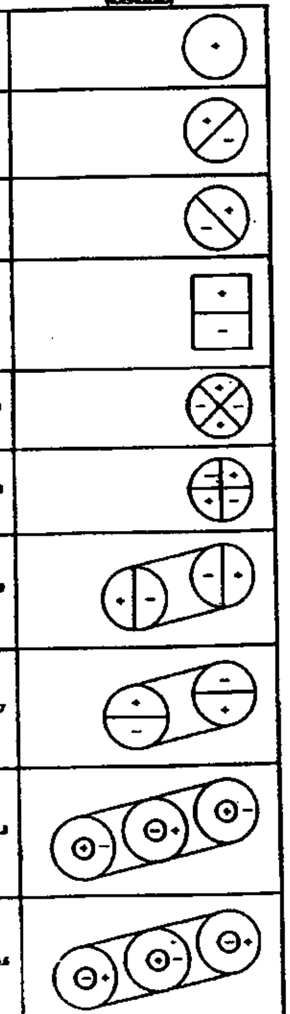


FIRST AXIAL

FLUX HARMONICS

MODE NUMBER	DESIGNATION	SUBCRITICALITY mk
0	FUNDAMENTAL	0
1	FIRST AZIMUTHAL-A	[illegible]
2	FIRST AZIMUTHAL-B	[illegible]
3	FIRST AXIAL	27.1
4	SECOND AZIMUTHAL-A	[illegible]
5	SECOND AZIMUTHAL-B	[illegible]
6	FIRST AZIMUTHAL-A X FIRST AXIAL	[illegible]
7	FIRST AZIMUTHAL-B X FIRST AXIAL	47.7
8	FIRST RADIAL X SECOND AXIAL-A	[illegible]
9	FIRST RADIAL X SECOND AXIAL-B	[illegible]

MODE SCHEMATIC (IDEALIZED)



Harmonics for CANDU (Cylindrical Reactor)

- **For homogeneous bare cylindrical reactor, flux shape given by (in r, θ , z co-ordinate):**

$\phi(r,\theta,z=J_M(\alpha_{ML}.r/R_0)*\cos(M*\theta)*\sin(N*\pi*z/H)$ M even

$\phi(r,\theta,z=J_M(\alpha_{ML}.r/R_0)*\sin(M*\theta)*\sin(N*\pi*z/H)$ M odd

where **J_M is the M^{th} order Bessel function,**

α_{ML} is the L^{th} zero of J_M

R_0 is the radius of the reactor

H is the height of the reactor

- **Various combinations of M and N give the Harmonics. Flux shape used as initial guess in *MONIC**

Harmonics - Natural Modes (Example)

1-D Problem Slab reactor, thickness from $x = -a/2 \text{ to} + a/2$

$$\frac{d^2\phi}{dx^2} + B^2\phi = 0$$

Ignore flux extrapolation beyond slab surface,

i.e. assume $\phi = 0$ at $x = a/2$ and $-a/2$

Note also symmetry $\phi = (x) = \phi(-x)$ and $\frac{d\phi}{dx} = 0$ at $x = 0$

Solution: $\phi(x) = A\cos Bx + C\sin Bx$

$\frac{d\phi}{dx} = 0$ at $x = 0$ **forces** $C = 0$

$\phi(\frac{a}{2}) = 0$ **forces** $\cos(\frac{Ba}{2}) = 0$

Therefore, $\phi(x) = A\cos(B_n x) = A\cos(\frac{n\pi}{a}x) \quad n = 1,3,5\ldots$

B_n are the eigenvalues, cos ($B_n x$) are the eigenfunctions (harmonics) $B_1 2$ is the buckling of the fundamental mode = $(\pi/a)^2$

Examples of Higher Harmonics - Natural Modes of a Slab Reactor

Steady State One-Group Diffusion Equation

$$D\nabla^2\phi - \Sigma_a\phi + s = 0$$

Define $L^2 = \frac{D}{\Sigma_a}$ (Unit cm^2)

Since $s = \eta\Sigma_{aF}\phi$ and $f = \Sigma_{aF}/\Sigma_a$

then $s = \eta f \Sigma_a\phi = k\infty\Sigma_a\phi$

$$\nabla^2\phi + \frac{k\infty - 1}{L^2}\phi = 0$$

Define $B^2 = \frac{k\infty - 1}{L^2}$

then $\nabla^2\phi + B^2\phi = 0$