ECOLE POLYTECHNIQUE

Introduction to Nuclear Reactor Kinetics

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by

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Introduction

- Reactivity is an important reactor physics concept useful to answer common questions such as:

  - What is a «critical» fission reactor?
  - How do you «start up» a reactor?
  - How does power vary with time when a reactor is subjected to a perturbation?
  - How can we control the power in a reactor?
  - What is meant by «prompt critical» or «super prompt critical»?
  - Can the reactor «explode» like a nuclear bomb? What about Tchernobyl?
  - ...

- **Reactivity characterizes *simply* the dynamic state of a nuclear reactor:**

  ⇒ *particularily useful in reactor control*

- Reactivity cannot be measured directly:

  ⇒ *obtained from neutron field equations defined over the entire reactor domain*

Course Objectives

- The basic interactions between neutrons and nuclei will be discussed

- The equations governing the neutron field in a reactor will be presented

- The formal definition of reactivity will be provided. Its reactor physics applications and limitations will be discussed.

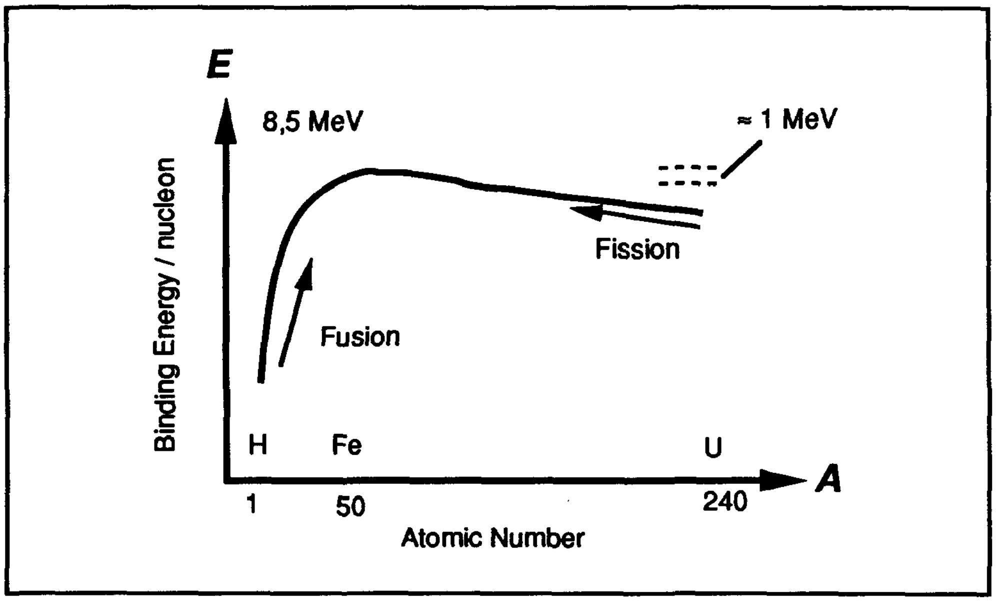
- The Point Kinetics equations will be derived and applied to typical situations in a CANDU reactor (as compared to other types of reactors).

- Numerical solutions with prompt temperature feedback will be presented

The Fission Process

- when nucleons are rearranged to a more stable form, we observe:
  - a mass defect (ΔM)
  - energy production ($E = \Delta M\, c^2$)

Average Binding Energy per Nucleon



$$\begin{bmatrix}\text{sum of}\\ \text{masses of}\\ \text{nucleons}\end{bmatrix} = \begin{bmatrix}\text{mass of}\\ \text{the bound}\\ \text{isotope}\end{bmatrix} + \frac{1}{c^2}\begin{bmatrix}\text{total}\\ \text{binding}\\ \text{energy}\end{bmatrix}$$

$$B_{avg}(A,Z) = \frac{\Delta M(A,Z)\, c^2}{A}$$

- fusion D-T:

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1 + \underbrace{\text{energy}}_{17.7\,MeV}$$

- fission U-235:

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{36}Kr^{95} + {}_{56}Ba^{139} + 2\ {}_0n^1 + \underbrace{\text{energy}}_{200\,MeV}$$

Fission

Energy Produced by Fission of U-235 (MeV)

Form	Energy produced	Recovered Energy	
		coolant	*moderator*
fission fragments:			
light	100	100	-
heavy	69	69	-
prompt neutrons	5	-	5
prompt γ	6	10*	5
(anti)neutrinos	11	-	-
fission product decay:			
β	7	7	-
γ	6	4	2
Total	*204 MeV*	*190 MeV*	*12 MeV*

* Including γ's from radiative capture.

- Similar values for other fissile isotopes (≈ 200 MeV, ± 5%)

- Fission of 1 kg of Uranium:

  - 950 MWd of energy
  - 13500 barrels of oil equivalent
  - 2830 Mg coal equivalent

Fission

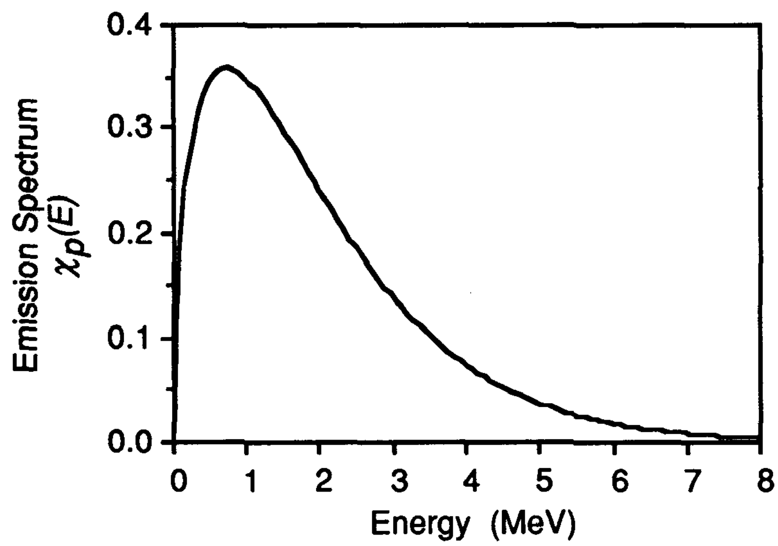
Number of prompt neutrons per fission

isotope	ν
U-233	2.438
U-235	2.430
Pu-239	2.871
Pu-241	2.969
U-238*	2.841

* Fast fission

- Slightly dependent on energy of incident neutron
- Fission is *isotropic*: direction of prompt fission neutrons is independent of direction of incident neutron.

Energy Spectrum of Prompt Fission Neutrons (U-235)



- *Average* energy of prompt fission neutrons (U-235) is 1.98 MeV

Neutron Interactions with Nuclei

- Notation ${}_Z X^A$

  X chemical symbol
  Z number of protons
  A number of nucleons (mass number)

- The Compound Nucleus

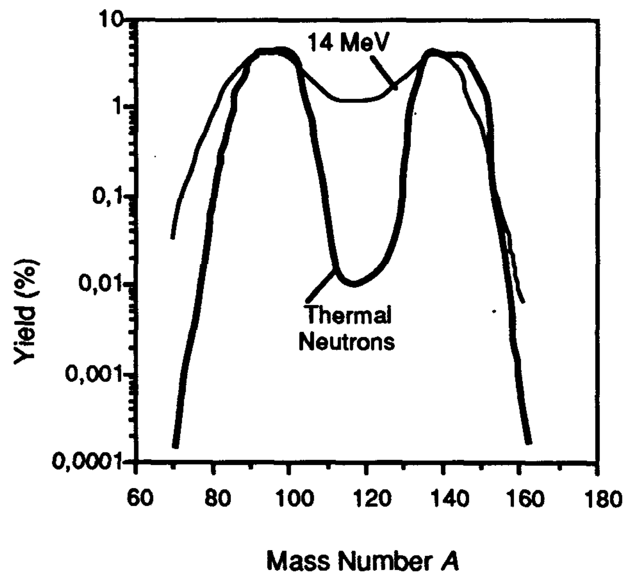
$$ {}_Z X^A + {}_0\mathrm{n}^1 \rightarrow \left({}_Z X^{A+1}\right)^* $$

- Decay of Compound Nucleus:

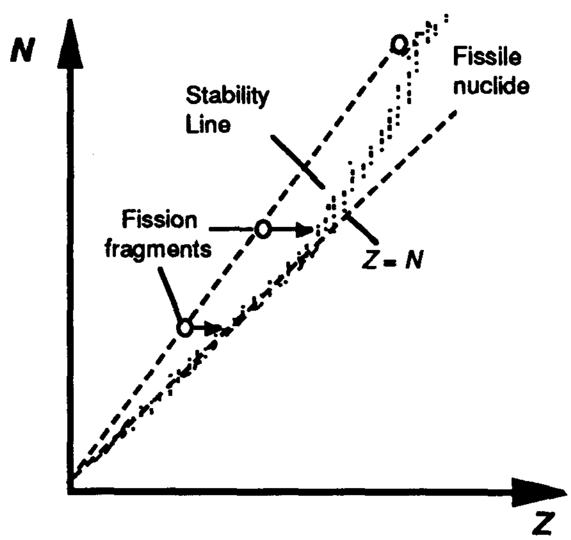
Elastic Scattering (n, n)	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_Z X^A + {}_0\mathrm{n}^1$
Inelastic Scattering (n, n')	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_Z X^A + {}_0\mathrm{n}^1 + \gamma$
Radiative Capture (n, γ)	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_Z X^{A+1} + \gamma$
Fission (n, f)	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_C PF_1{}^B + {}_{Z-C} PF_2{}^{A+1-B-\nu} + \nu\, {}_0\mathrm{n}^1 + \gamma$
(n, p)	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_{Z-1} Y^A + {}_1\mathrm{H}^1$
(n, α)	$\left({}_Z X^{A+1}\right)^* \rightarrow {}_{Z-2} Y^{A-3} + {}_2\mathrm{He}^4$
(n, 2n), (n, 3n)...	–

Fission Product Distribution

- yield



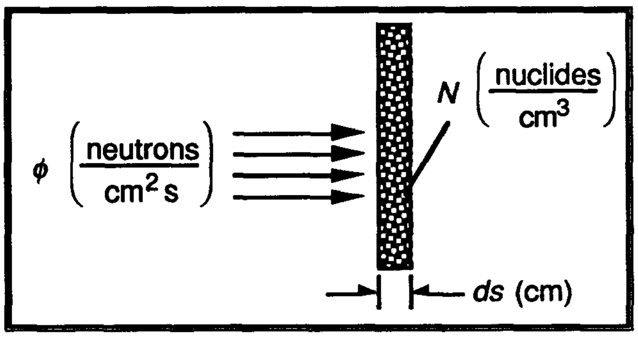
- stability



Neutron Cross Sections and Reaction Rates

	Symbol	units
neutron angular density	$n(\vec{r},E,\vec{\Omega},t)$	[n/cm^3/eV/st]
neutron velocity	$\vec{v} = v\,\vec{\Omega}$	[cm/s]
angular flux density	$\Phi(\vec{r},E,\vec{\Omega},t) = n(\vec{r},E,\vec{\Omega},t)\cdot v(E)$	[n/cm^2/s/eV/st]
scalar flux density	$\phi(r,E,t) = n(\vec{r},E,t)\cdot v(E)$	[n/cm^2/s/eV]
scalar flux (in group *g*)	$\phi_g(r,t) = \int_{E_{g-1}}^{E_g} \phi(r,E,t)\,dE$	[n/cm^2/s]
microscopic cross section	$\sigma(E)$	[cm^2]
macroscopic cross section	$\Sigma(\vec{r},E,t) = N(\vec{r},t)\cdot\sigma(E)$	[cm^{-1}]
reaction rate (per unit volume)	$R = \Sigma\cdot\phi$	[s^{-1}cm^{-3}]

<u>Ex</u>: for a reaction of type *x*, cross sections are determined experimentally by measuring the attenuation of an incident neutron beam (of given *E*). If the volume contains many isotopes (i), simply sum over i to obtain the total reaction rate:



$$dR_x = \sigma_x\,\phi\,N\,ds$$

Integrating over 1 cm^3:

$$R_x = (N\,\sigma_x)\,\phi$$

$$= \Sigma_x\,\phi$$

Neutron cross sections

Cross sections for thermal neutrons (0,025 eV), in barns (Westcott, 1960)

Isotope	σ_f	σ_γ	σ_a
U^{233}	525.0	53.0	578.0
U^{235}	577.0	106.0	683.0
U^{238}	–	2.70	2.70
Pu^{239}	742.1	286.9	1029.1
Pu^{240}	–	277.9	277.9
Pu^{241}	1015.2	381.0	1396.2

Neutron scattering

Isotope	*A*	*N*
Hydrogène (H^1)	1	18
Deuterium (H^2)	2	25
Graphite (C^{12})	12	115
Uranium (U^{238})	238	2172

N = number of collisions required to slow down 2 MeV neutrons to thermal energies (0.025 eV) (Weinberg,1958).

The Neutron Chain Reaction

- Neutron cross sections :
  - most isotopes possess relatively small absorption cross sections ($\sigma_a = \sigma_f + \sigma_c$) of the order of 1-10 barns. Mean free path of neutrons in matter is large, of the order of cm's.
  - notable exception: heavy elements (fuel), with a large absorption cross section ($\sigma_a \approx 1000$ b)
  - only odd-number isotopes are fissile for thermal neutrons (U-233, U235, Pu-239, Pu-241, ...)
  - even numbered isotopes allow fast fission (threshold energy ≈ 1 MeV)

- Fission neutrons appear with an energy ≈ 1 MeV, where fission cross section is small compared to thermal energies (by a factor of 1000)

  ⇒ *advantage in slowing down neutrons (via scattering) in moderator material*

  ⇒ *thermal reactors*

- To maintain the chain reaction in thermal reactors, neutrons must be slowed down over 8 decades, while avoiding resonance absorption in the heavy element (notably U-238, a major constituant of the fuel)

  ⇒ *resonance capture is limited by self-shielding in fuel pins*

  ⇒ *heterogeneous reactors*

- *A controlled fission chain reaction will largely depend on a critical mass of fuel and moderator, and is very sensitive to geometry and the distribution of neutrons in the reactor (space and energy distribution)*

  ⇒ *neutron field*

Delayed Neutrons

- certain fission products are *precursors* to the emission of delayed neutrons. For example, approximately one fission in a thousand in U-235 yields Br-87 as fission product. This isotope is radioactive and decays into an excited state of Kr-87, which stabilizes into Kr-86 by emitting a neutron:

$$\underbrace{{}_{35}Br^{87}}_{\substack{\text{fission}\\ \text{product}\\ (\textit{precursor})}} \xrightarrow[(55\,s)]{} {}_{36}Kr^{87} + \underbrace{{}_{-1}e^{0}}_{\beta^{-}\ \textit{radiation}}$$

$$\underbrace{{}_{36}Kr^{87}}_{(\textit{emitter})} \xrightarrow[(10^{-14}\,s)]{} {}_{36}Kr^{86} + \underbrace{{}_{0}n^{1}}_{\substack{\text{«delayed»}\\ \text{neutron}}}$$

- The production of this neutron in the fuel is «delayed» with regards to the fission event due to the delay associated with radioactive decay of the precursor. The half-life of precursors varies between 0.2 s and 55 s.

- Relatively few fission products are delayed neutron precursors. The *delayed neutron fraction* is defined as:

$$\boxed{\beta = \frac{\nu_d}{\nu} = \frac{\text{delayed neutrons resulting from a single fission event}}{\text{total number of fission neutrons (prompt + delayed)}}}$$

Delayed Neutrons (cont'd)

- We note that the delayed neutron fraction is small and is specific to the fissioning nuclide. It varies significantly from one isotope to the other:

	ν_d	β
U-233	0.0067	0.0027
U-235	0.0166	0.0068
Pu-239	0.0065	0.0023
Pu-241	0.0154	0.0052
*U-238**	0.0450	0.0158

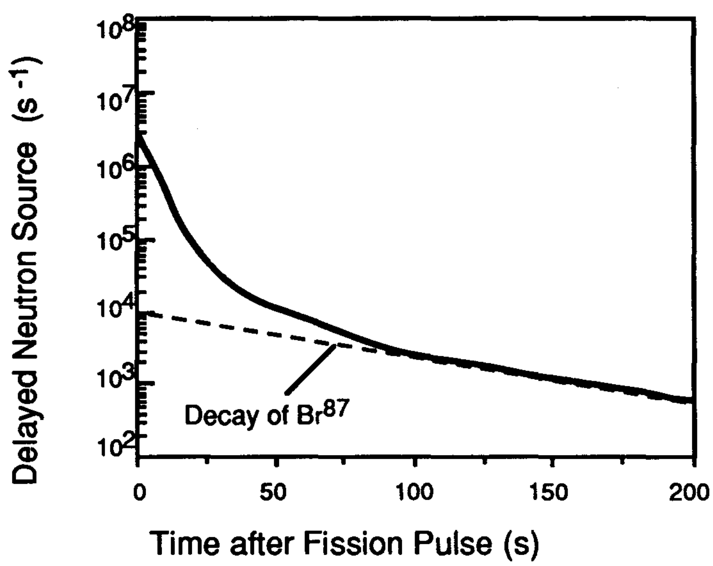
* Fast fission

- Although small, the delayed neutron fraction plays an extremely important role in Reactor Kinetics, as we shall see in this course.

- It will be important to evaluate the effective average β for a given reactor, considering the fission rate in each fissioning isotope.

- For each fissile isotope, delayed neutron precursor properties can be obtained experimentally, with a curbe fit over a limited number of delayed groups (generally 6 for the fuel):

$$S_{di}(t) = n_{fi} \sum_{k=1}^{6} \nu_{dki} \lambda_{ki} e^{-\lambda_{ki} t}$$



Decay Constants for the Delayed-Neutron Groups, λ_k
(Ott, 1985)

Group	λ (s^{-1})	Half-life (s)
1	0.0129	53.73
2	0.0311	22.29
3	0.134	5.17
4	0.331	2.09
5	1.26	0.55
6	3.21	0.22

Delayed Neutrons Groups

- A single set of decay constants can be used for all isotopes:
- The following values were obtained from measurements:

Fraction of Delayed Neutrons from the Fuel, β_{kl} (%)

Group	^{235}U	^{239}Pu	^{241}Pu	^{238}U
1	0.0251	0.0087	0.0066	0.0206
2	0.1545	0.0639	0.1234	0.2174
3	0.1476	0.0493	0.0932	0.2570
4	0.2663	0.0747	0.2102	0.6156
5	0.0756	0.0235	0.0981	0.3570
6	0.0293	0.0080	0.0086	0.1190
Total β	0.6984	0.2281	0.5389	1.587

Photoneutrons

- In heavy water reactors (CANDU), additional delayed neutrons appear in the moderator, called *photoneutrons*. They constitute only ≈ 5% of β but have a much longer half-life (up to 15 days):

$$\underbrace{\gamma}_{\substack{\gamma \text{ radiation from} \\ \text{fission product} \\ (E_\gamma \geq 2.2\text{ MeV})}} + \underbrace{{}_1H^2}_{\substack{\text{deuterium in} \\ \text{moderator}}} \rightarrow {}_1H^1 + \underbrace{{}_0n^1}_{\substack{\text{«delayed»} \\ \text{neutron}}}$$

- The half life of the photoneutron precursors is generally longer. 9 groups are required:

Fraction of Delayed Neutrons from the Moderator (Photoneutrons), β_k(%) (Kugler, 1975)

Group	$T_{1/2}$	λ_k (s^{-1})	β_k (%)
7	12.8 d	6.26×10^{-7}	1.653×10^{-5}
8	2.21 d	3.63×10^{-6}	3.372×10^{-5}
9	4.41 h	4.37×10^{-5}	1.058×10^{-4}
10	1.65 h	1.17×10^{-4}	7.670×10^{-4}
11	27 min	4.28×10^{-4}	6.779×10^{-4}
12	7.7 min	1.50×10^{-3}	1.101×10^{-3}
13	2.4 min	4.81×10^{-3}	2.298×10^{-3}
14	41 s	1.69×10^{-2}	6.694×10^{-3}
15	2.5 s	2.77×10^{-1}	2.136×10^{-2}
TOTAL (photoneutrons)	16.7min*	6.92×10^{-4}	3.306×10^{-2}

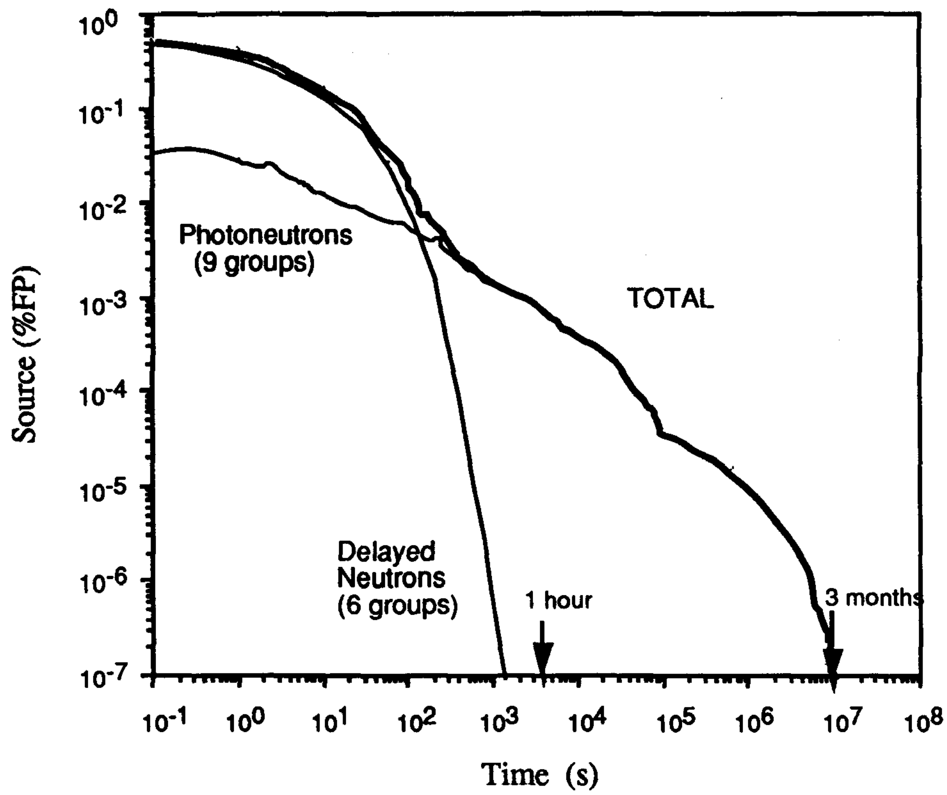
*Average values: $\lambda_{ph} = \Sigma\,(\beta_k/\lambda_k)/\,\Sigma\,\beta_k$ and $T_{1/2} = \ln 2 \cdot \lambda_{ph}$

Delayed Neutron Source in a CANDU

- As a result, ther exists a slowly decreasing neutron source in a reactor after shutdown.

- In a CANDU, the photoneutrons dominate after only 5 minutes, eventhough at the beginning, they constitute only 5% of the delayed source.

Delayed Neutron Source in a CANDU
(after shutdown from 100%FP)



Fuel Burnup

- Fuel irradiation (or exposure) is defined by:

$$\omega(t) = \int_0^t \phi_c \, dt'$$

where ϕ_c is the neutron flux *in the fuel.*

- The irradiation unit is the *neutron/kilobarn*

$$\frac{n}{kb} = \left\{ \underbrace{\frac{n}{cm^2 - sec}}_{(\phi)} \cdot \underbrace{sec}_{(T)} \right\} \times \underbrace{10^{-24} \frac{cm^2}{barn} \times 10^3 \frac{barn}{kilobarn}}_{10^{-21}}$$

- Fuel burnup quantifies the total *fission energy* produced in a fuel element per unit mass of initial *heavy elements* (uranium) in the fuel, up to time t:

  - *energy produced*

$$E_k(t) = \int_0^t \underbrace{p_{k(t)}}_{\substack{fission \\ power}} dt' = \int_0^t \kappa \underbrace{\Sigma_f \phi V_k}_{\substack{fission \\ rate}} dt'$$

$$= \bar{p}_k \cdot T$$

  - *units*

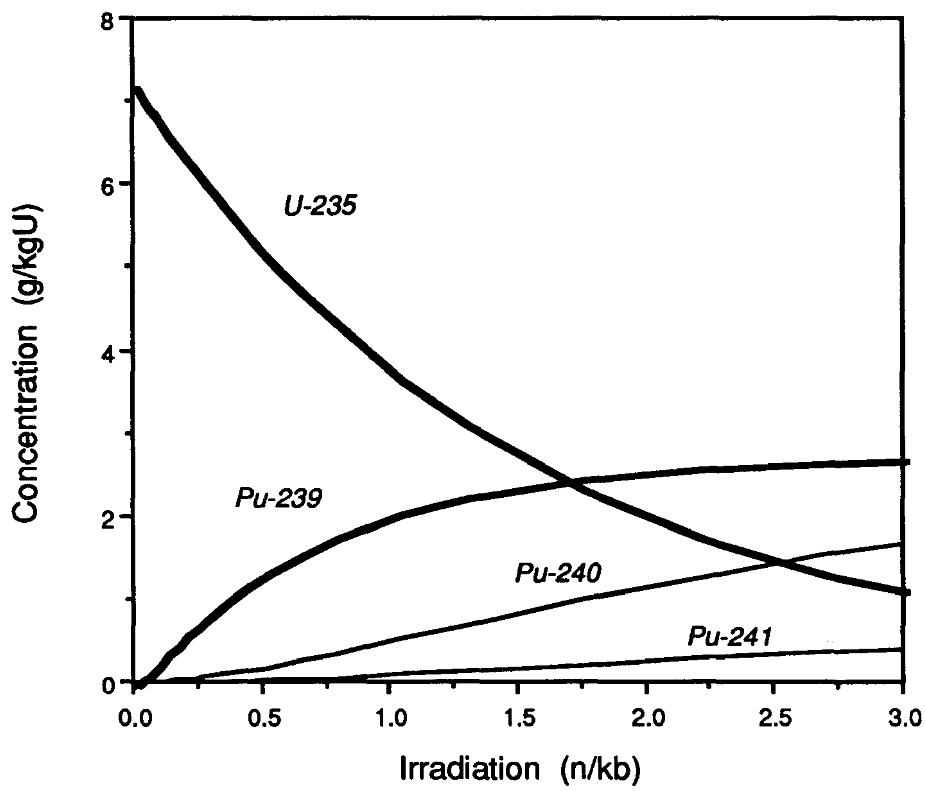
$$E = (\underbrace{kW}_{(p)} \cdot \underbrace{sec}_{(T)}) \times \left(10^{-3} \frac{MW}{kW}\right)\left(\frac{1}{3600} \frac{h}{sec}\right)$$

$$= MWh$$

$$B_k = \frac{E_k}{m_k} \Rightarrow \left(\frac{MWh}{kg}\right)$$

Fuel Composition in a 37 element CANDU Fuel Bundle producing 690 kW

Temps (jours)	*irradiation (n/kb)*	*burnup MWh/kg)*	*U-235 (g/kgU)*	*Pu-239 (g/kgU)*	*Pu-241 (g/kgU)*
0.	0.	0.00	7.200	0.000	0.000
10.	.093	8.69	6.782	0.225	0.000
30.	.273	26.09	6.038	0.771	0.003
50.	.446	43.46	5.396	1.185	0.012
75.	.658	65.21	4.700	1.575	0.032
90.	.784	78.26	4.329	1.759	0.047
120.	1.040	104.33	3.672	2.043	0.085
150.	1.300	130.42	3.111	2.246	0.127
200.	1.749	173.90	2.353	2.467	0.204
250.	2.217	217.37	1.769	2.596	0.277
300.	2.692	260.85	1.325	2.672	0.345



ASSIGNMENT 1

Consider the two following nuclear reactions:

$^{3}H + {}^{2}H \rightarrow {}^{4}He + n + E$

$^{235}U + n \rightarrow {}^{139}Xe + {}^{94}Sr + 3n + E$

Using the information below:

a) Calculate the energy E produced in each reaction
b) Indicate which of these reactions produces more energy if the same mass of reactants is used.

Information:

Atom (or particle)	Mass (AMU)
^{3}H	3.016
^{2}H	2.014
^{4}He	4.002
^{235}U	235.044
^{139}Xe	138.918
^{94}Sr	93.915
n	1.009

1 AMU = 931.5 MeV
= 1.66×10^{-27} kg

1 MeV = 1.60×10^{-13} J

ASSIGNMENT 2

Boron and Godolinum are two soluble poisons that can be introduced in the moderator to control the reactivity in a CANDU reactor. The properties of these poisons are given in the attached table.

a) Indicate which isotopes of boron and gadolinium constitute the neutron poison.

b) A natural boron concentration of 2.15×10^{17} at /cm^3 is required in the moderator to introduce -28 mk of reactivity (simulating the equilibrium Xenon load). Calculate the equivalent gadolinium concentration which will produce the same reactivity effect.

Isotope	Proportion (%)	Microscopic thermal neutron absorption cross section (barns)	Type of reaction
Bore			
B-10	19.8	3840	(n,α)
B-11	80.2	.005	(n,γ)
Gadolinium			
Gd-152	.2	10	(n,γ)
Gd-154	2.1	80	(n,γ)
Gd-155	14.8	6.1×10^4	(n,γ)
Gd-156	20.6	2	(n,γ)
Gd-157	15.7	2.5×10^5	(n,γ)
Gd-158	24.8	2.4	(n,γ)
Gd-160	21.8	.8	(n,γ)
Atomic weight	Boron:	10.8	
	Gadolinium:	157.2	

ASSIGNMENT 3

A natural UO_2 fuel bundle is removed from a reactor following an exposure to an average thermal neutron flux of 1.0×10^{14} neutrons $cm^{-2}s^{-1}$ for 200 days.

a) Calculate the burnup of the fuel in n/kb.

b) Calculate the U^{235} content that remains after this exposure, expressed in g/kgU.

c) If the concentrations of Pu^{239}, and Pu^{241} at this time are 2.39 g/kgU and .221 g/kgU, respectively, calculate the fraction of the total number of fissions in the fuel that were due to plutonium (both Pu^{239} and Pu^{241}) just before discharge from the reactor.

(Neglect the difference in atomic weights between U^{235}, Pu^{239} and Pu^{241}).

The table below provides the data required for answering this question.

	σ_f	$\sigma_{n,\gamma}$
U^{235}	580.2 b	98.3 b
Pu^{239}	741.6 b	271.3 b
Pu^{241}	1007.3 b	368.1 b

1 barn = $10^{-24}cm^2$

fresh fuel composition: U^{235} = 7.2 g/kgU

U^{238} = 992.8 g/kgU

ASSIGNMENT 4

If on the average, 200 MeV of energy is produced per fission, calculate the fuel consumption per year in a CANDU-600 reactor producing 2064 MW in the primary coolant.

The exit fuel burnup actually acheived is 180 MWh/kg. Is this value consistent with your previous evaluation? If not, why is it different?

ASSIGNMENT 5

Assume that 3% of all fissions in a CANDU reactor are due to fast fission in U-238. Using the data provided in the class room, calculate the total delayed neutron fraction and an appropriate average decay constant for a single delayed neutron group:

a) for a reactor containing only fresh fuel (natural uranium, 0.711%, U-235)

b) for a reactor at equilibrium refuelling, with an average exit burnup of 185 MWh/kg.