

Fluid Mechanics - Course 123

HYDRODYNAMIC FILM LUBRICATION

The theory of hydrodynamic film lubrication depends upon laminar flow criteria. Initially, it may seem unlikely that laminar flow could ever occur in a journal bearing because of the tangential velocity of the journal. The answer lies in the fact that the oil film is relatively thin and the viscous forces are still dominant in controlling the oil film behaviour. In large journal bearings for turbomachinery the oil film thickness is about 0.13 millimeters. In this type of oil film the values of Reynolds number are well below the critical values for laminar flow.

The slipper bearing is illustrated on page 6 and consists of two surfaces that are angled relative to each other, one fixed and the other moveable, and separated by a fluid film.

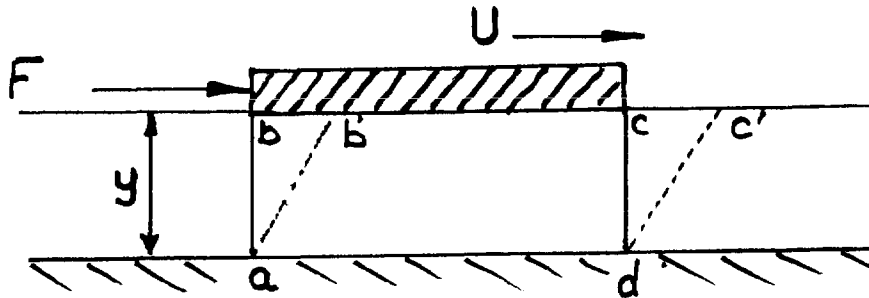
The slipper bearing and the journal bearing develop their pressure forces by the same mechanism, the only major difference being that the bearing surface of the journal bearing is curved.

In a conventional analysis for the calculation of ideal bearing performance, the following assumptions are made:

- a) Film thickness is small compared with bearing dimensions.
- b) The inertia of the oil film is negligible.
- c) The oil film is unable to sustain sub-atmospheric pressure.
- d) Oil pressure is atmospheric at the supply and drain boundaries from the bearing.
- e) Laminar flow occurs in the pressure-bearing film.
- f) Oil is a simple Newtonian fluid with viscosity independent of shear rate.
- g) Viscosity and density are constant throughout the bearing.

In hydrodynamic lubrication, large forces are developed in small clearances, when the surfaces are slightly convergent and one surface is in motion so that the fluid is "wedged" into the decreasing space.

By definition a fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be. A shear force acts tangentially to the surface considered and when divided by the surface area provides the average shear stress.



Consider the illustration of a fluid between two parallel plates as shown. The bottom plate is stationary and the upper plate is allowed to move at steady velocity U when a force F is applied to the upper plate.

The fluid in immediate contact with a solid boundary has the same velocity as the boundary, ie, there is no slip at the boundary. As the upper plate moves, the fluid flows to a new position, each particle moving parallel to the plate and the velocity varying uniformly from zero, at the stationary plate to U at the upper plate.

In a Newtonian fluid there is a linear relationship between the magnitude of applied shear stress and the rate of angular deformation. For practical purposes oils may be considered to be Newtonian fluids.

The force required to move the plate depends directly upon the velocity U , the sectional area of the film and inversely upon the film thickness y .

Thus in equation form:

$$F = \mu \frac{AU}{y}$$

Where μ is the proportionality factor and applies only to a particular fluid, μ is of course the 'dynamic viscosity' of the fluid.

If we substitute $T = F/A$ for shear stress then the equation becomes:

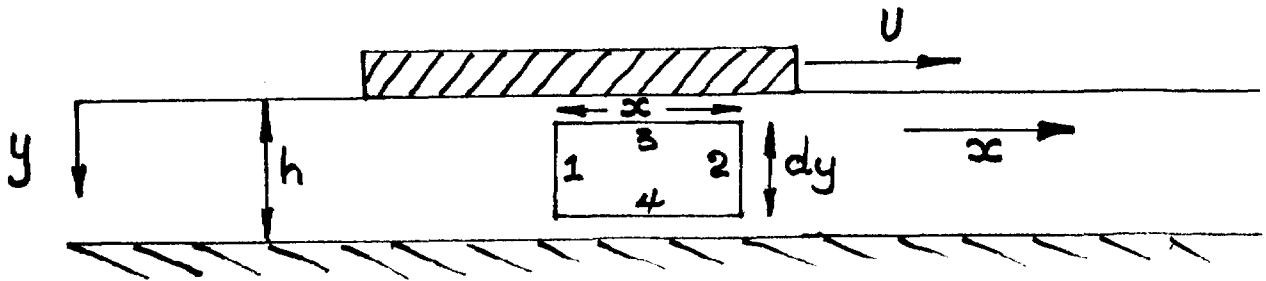
$$T = \mu \frac{U}{y}$$

The ratio $\frac{U}{y}$ is the angular velocity of the line ab or it is the rate of angular deformation of the fluid. Thus in differential form $T = \mu \, du/dy$.

The pressure distribution and the position of the centre of pressure in a hydrodynamic oil film are of paramount importance when looking at slipper and journal bearings where oil is wedged to produce a pressure bearing film.

In deriving an expression for pressure distribution, one dimensional, incompressible, steady, laminar flow is considered and leakage is initially considered negligible.

Consider, again the moving plate and let us examine the forces acting on an elemental area.



Choosing a co-ordinate system, x is in the direction of motion and the y axis is defining a point at any depth in the fluid film. An element of lubricant is selected having length dx , depth dy and width in the z direction of dz . Initially we must consider the forces acting on this element which we may determine by multiplying the pressure that exists, on the element surface, by the area of the surface.

Consider the four elemental surfaces:

Surface 1

The pressure at this point is P and acts normal to the surface dy . The force produced is thus Pdy in the direction of motion which is considered positive.

Surface 2

The pressure at this surface has increased to a new value which depends upon the distance x from P . Thus the increased pressure is $P + \frac{dP}{dx} \cdot dx$ which acts normal to the area dy , thereby creating a force in opposition to the direction of motion and is, therefore, negative. Thus force is:

$$\left(P + \frac{dP}{dx} \cdot dx \right) dy$$

Surface 3

The shear force on the top surface of the element is acting against the direction of motion and is therefore negative. Shear force = Shear Stress \times area in shear.

$$\text{Thus } F_s = S_s \times dx$$

Surface 4

The shear stress increases as the film thickness increases and as x increases, and recognizing this double dependence, we must use the partial derivative $\partial S_s / \partial y$. The shear stress with increased depth dy is now $\left(S_s + \frac{\partial S_s}{\partial y} dy \right)$ and the shear force is $\left(S_s + \frac{\partial S_s}{\partial y} dy \right) dx$ in the positive direction.

Finally, in an equilibrium condition where there is no acceleration the sum of the elemental forces must be zero.

$$\text{Thus } Pdy - \left(P + \frac{dP}{dx} dx \right) dy + \left(S_s + \frac{\partial S_s}{\partial y} dy \right) dx - S_s dx = 0$$

$$\therefore Pdy - Pdy - \frac{dP}{dx} \cdot dx \cdot dy + S_s dx + \frac{\partial S_s}{\partial y} dy \cdot dx - S_s dx = 0$$

$$\text{Hence } \frac{dP}{dx} \cdot dx \cdot dy = \frac{\partial S_s}{\partial y} \cdot dy \cdot dx$$

$$\therefore \frac{dP}{dx} = \frac{\partial S_s}{\partial y}$$

If we re-examine our definition of viscosity where

$$\mu = \frac{\text{Shear Stress}}{\text{rate of shear strain}} = \frac{S_s}{\partial U / \partial y}$$

We see that:

$$S_s = \mu \partial u / \partial y$$

Again the partial derivative is used because U depends upon both x and y.

If we substitute this expression for shear stress into the pressure equation we now have:

$$\frac{dP}{dx} = \mu \partial^2 u / \partial y^2$$

$$\partial^2 u / \partial y^2 = \frac{1}{\mu} \frac{dP}{dx}$$

If we now hold x constant and integrate this expression twice with respect to y, we can determine the velocity distribution through the fluid film at a point x.

Thus:

$$\partial u / \partial y = \frac{1}{\mu} \frac{dP}{dx} y + C_1$$

and
$$u = \frac{1}{2\mu} \frac{dP}{dx} y^2 + C_1 y + C_2$$

We may evaluate the integration constants by assuming that there is no slip between the oil and the boundary surfaces. Thus when $y = 0$, $u = U$ and when $y = h$, $u = 0$.

Thus $C_2 = U$

and $C_1 = \frac{-h}{2\mu} \frac{dP}{dx} - \frac{U}{h}$

and substituting:

$$u = U \frac{(h-y)}{h} - y \frac{(h-y)}{2\mu} \frac{dP}{dx}$$

which may be rearranged as follows:

$$u = U \left(1 - \frac{y}{h(x)}\right) - \frac{h^2(x)}{2\mu} \frac{dP}{dx} \frac{y}{h(x)} \left(1 - \frac{1}{h(x)}\right)$$

In the flow of the fluid film the continuity of mass must be satisfied and the mass flow at every section must be constant.

Thus $Q_m = \rho \times Q_v = \rho \times A \times V$

thus $Q_m = \rho \int_0^h u \, dy = \text{constant}$

Substituting the velocity expression for u we get

$$Q_m = \rho \int_0^{h(x)} \left[U \left(1 - \frac{y}{h(x)} \right) - \frac{h^2(x)}{2\mu} \frac{dP}{dx} \frac{y}{h(x)} \left(1 - \frac{y}{h(x)} \right) \right] dy$$

which gives

$$Q_m = \rho \left(\frac{Uh(x)}{2} - \frac{h^3(x)}{12\mu} \frac{dP}{dx} \right)$$

We now have an expression which allows the pressure distribution to be evaluated at every point with respect to x .

Re-arranging the equation.

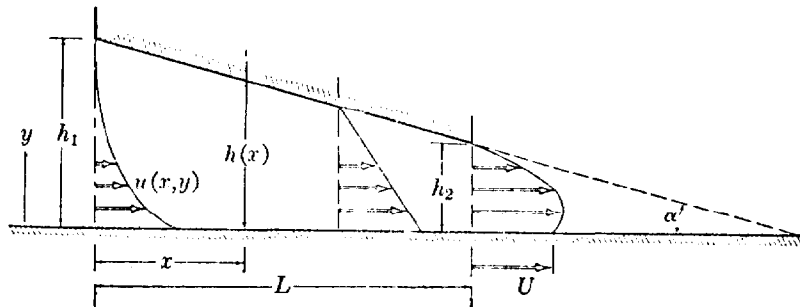
$$\frac{dP}{dx} = 12\mu \frac{U}{2h^2(x)} - \frac{Q}{h^3(x)\rho}$$

This is a total differential equation which upon integration gives the pressure distribution with respect to x .

Thus

$$P(x) = P_0 + 6U\mu \int_0^x \frac{dx}{h^2(x)} - \frac{12Q\mu}{\rho} \int_0^x \frac{dx}{h^3(x)}$$

In practice we are unlikely to know the maximum and minimum values of $h(x)$ or the value of length over which the change occurs.



The Slipper Bearing

In the diagram we can see that if h is a linear function of x that:

$$h(x) = h_1 - \frac{h_1 - h_2}{L} x$$

By substituting and integrating the expression for $P(x)$ we may see that the pressure at point x is given by the expression:

$$P - P_0 = \frac{6\mu Ux (h - h_2)}{h^2 (h_1 + h_2)}$$

(The mathematics are tedious and of no direct benefit, however, if the approach is of interest, the exercise is briefly as follows: Boundary condition states that $P = P_0$ when $x = L$, substitute into $P(x)$ equation and re-arrange for Q . Substitute:

$$h(x) = h_1 - \frac{h_1 - h_2}{L} x$$

into the expression for Q and then use this value of Q in the original Px equation.)

We can see that even when the dynamic viscosity is relatively small, very high film pressures may be developed if the mean thickness of the oil film is small.

Although our initial equations were developed for slider bearings, they may be applied to journal bearings by using the assumption that curvature, on an elemental basis, is to be neglected. This implies that the films are to be developed so as to cause them to lie flat.

If the angle of inclination α of the slipper is known, the pressure distribution may be expressed as a dimensionless ratio. From the pressure equation the dimensionless pressure distribution is

$$\frac{L(P - P_0)}{6\mu U} = \frac{Lx(h - h_2)}{h^2 (h_1 + h_2)}$$

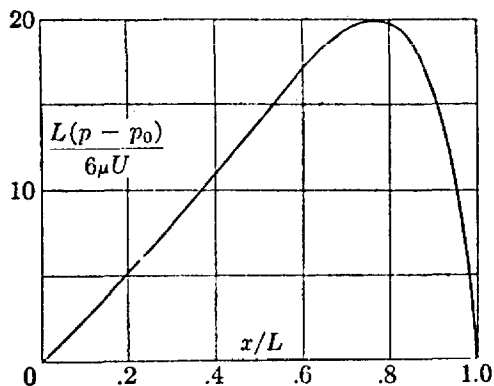
Expressing h in terms of $\tan \alpha$ the equation becomes:

$$\frac{L(P - P_0)}{\mu V} = \frac{\frac{x}{L} \tan \alpha (1 - \frac{x}{L})}{(\frac{h_1}{L} - \frac{x}{L} \tan \alpha)^2 (\frac{h_1 + h_2}{L})}$$

If α was 5° and the dimensionless ratio $\frac{h_1}{L} = 0.15$, then the ratio $\frac{h_2}{L}$ can be deduced to be 0.0449. By substitution the expression becomes:

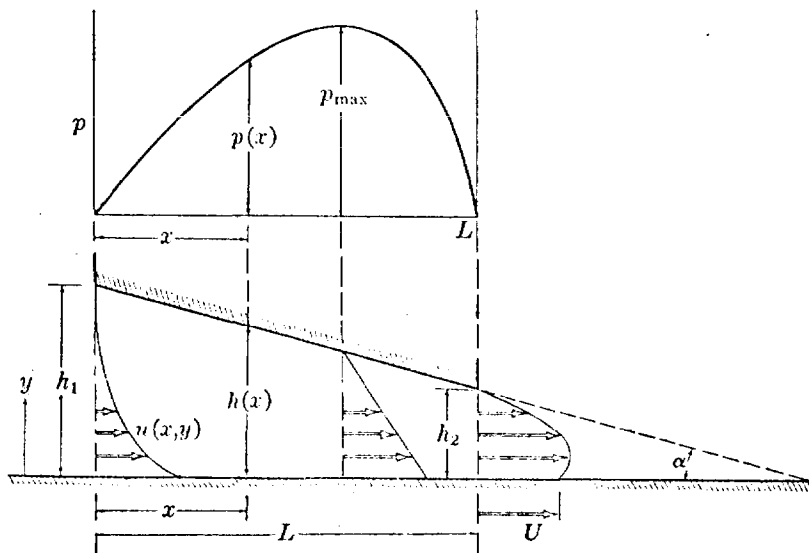
$$\frac{L(P - P_0)}{\mu U} = \frac{0.538 \frac{x}{L} (1 - \frac{x}{L})}{(0.15 - 0.1051 \frac{x}{L})^2}$$

If this expression is plotted as a function of the dimensionless bearing length x/L we obtain a pressure distribution as shown in the diagram.



Pressure distribution for $\alpha = 5^\circ$ and $h_1/L = 0.15$.

Returning to our original slipper bearing we can see how this pressure distribution is related.



The slipper bearing.

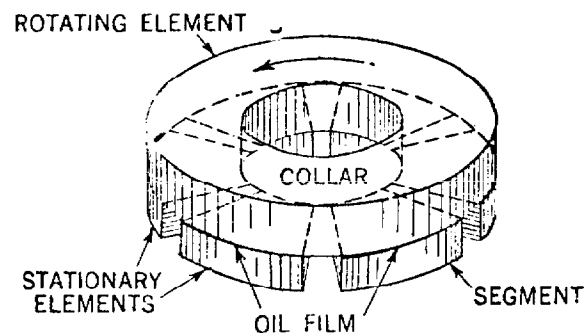
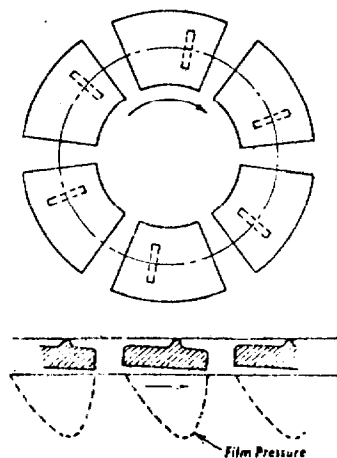
At both ends of the slipper the pressure is P_0 . As we progress along the bearing, the pressure increases to a maximum value and then decreases to P_0 . The point at which the maximum pressure is generated is neither the thinnest oil film section nor at the centre of the bearing. The maximum

pressure and its location may be determined by differentiating P with respect to x and setting the result $\frac{dP}{dx} = 0$.

As the angle of the slipper increases the centre of pressure moves further away from the geometric centre until at the maximum possible inclination, the 'heel' of the slipper touches the bearing plate when the centre of pressure is at the heel. The pressure at this point is then infinite. However under these conditions flow between the slipper and plate is impossible and so this limiting case has no physical significance.

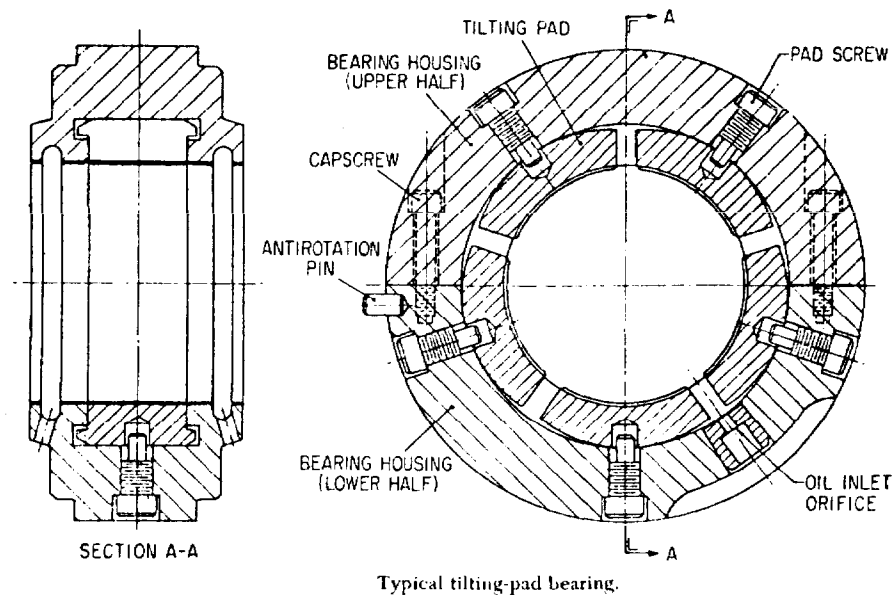
This unwrapped or developed oil film is typical of that used in a Michell or Kingsbury bearing which uses a series of pads which are tilted so that the wedging action is produced.

This type of bearing may be a thrust bearing with the pads running against a collar on the shaft:



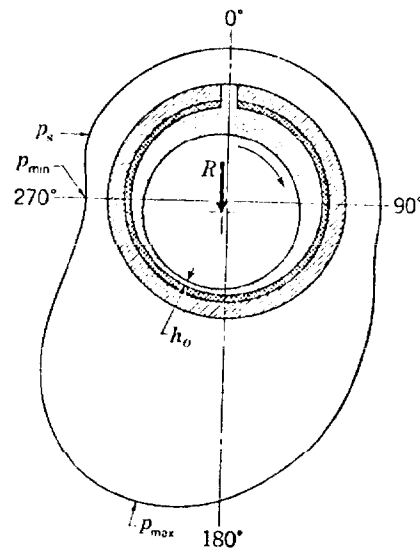
Basic elements of the Kingsbury thrust bearing, showing the wedge-shaped oil films.

it may be a journal bearing where the bearing surface is discontinuous and consists of several pads as shown.



We can see that the tilted pad bearing uses a number of bearing surfaces which produce centres of pressure having lower maximum values of pressure than would be the case in a continuous bearing surface. The tilted pad journal bearing cannot carry loads as high as a continuous surfaced bearing but does have the advantage that closer control of radial shaft movement may be obtained if it is not possible to use antifriction bearings.

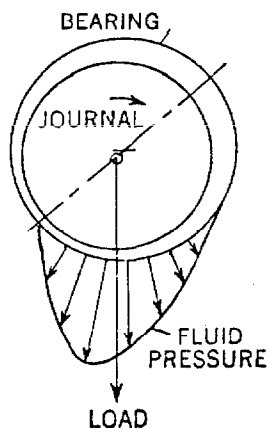
The pressure distribution on a continuously surfaced journal bearing is a wrapped distribution of the pressure as discussed for the plain slipper bearing.



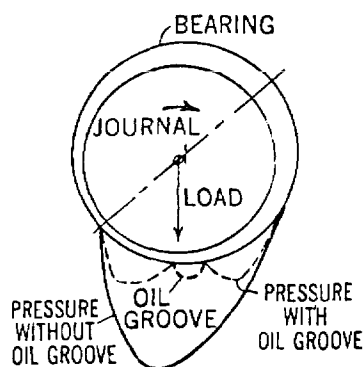
Distribution of oil pressure in a journal bearing.

In this diagram we can see that the minimum oil thickness h_0 occurs not at the bottom of the journal but is displaced clockwise from the bottom as shown. This can be explained by the fact that a positive oil film pressure develops in the converging half of the film and reaches a maximum somewhere between the bottom of the bearing and the location of h_0 .

The presence of oil grooves for supply or drainage may radically change the pressure distribution. This may be seen in the following figure which illustrates the reduction in pressure arising from the discontinuity of bearing surface created by an oil groove.

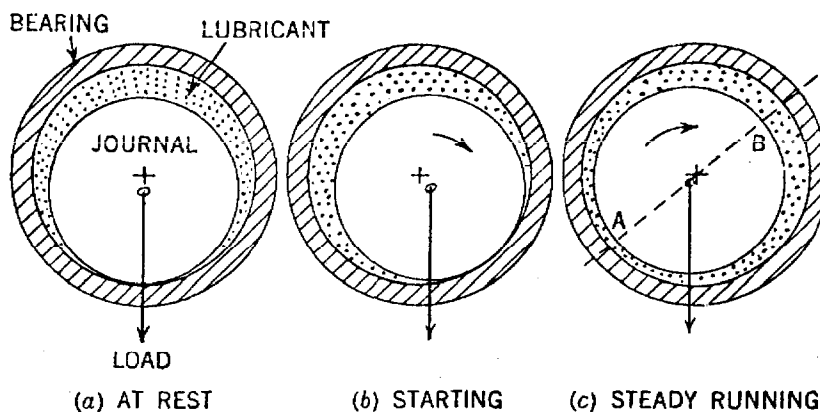


Pressure distribution around the circumference of a bearing—radial ordinate represents positive pressure.



Effect of oil groove on film pressure.

Another topic of significance is the change of attitude of the journal centre with respect to the bearing centre.



Cylindrical or journal bearing.

Looking at illustration (a) we can see that when the shaft is at rest the journal centre is immediately below the bearing centre.

As the journal starts to rotate there is no established oil film separating the journal and bearing and the bearing may be considered 'dry' in a hydrodynamic sense. The effect causes the journal to climb up the right-hand side of the bearing and the equilibrium position is obtained when the frictional force is balanced by the tangential component of the load. This change of attitude is described by illustration (b).

As the journal speed increases, the pumping action of the rotating journal increases and pumps oil round the bearing in a clockwise direction thereby establishing the load bearing film. The wedging action of the film causes the journal to rise and move across to the left of the bearing. This final running position is shown in illustration (c).

Normally the load on the bearing is downwards and the pressure film is developed in the bottom half of the bearing and oil supply is to the top half of the bearing. In the relatively unusual situation where the bearing load is upwards, failure to recognize that the introduction of oil supply to the top half of the bearing will radically reduce the bearing oil film pressures, could have disastrous results.

It should be realized that if the journal centre was coincident with the bearing centre then no support bearing oil pressure could be generated because there would be no wedging action. Further discussion of journal bearings may be found in the 122 Mechanics course.

We have assumed so far that the side leakage is negligible. In practice the side leakage has to be sufficient for cooling flow but not great enough to destroy the bearing film pressure. The pressure distribution across a section of the oil film shows that the pressure rises from the supply pressure to the film pressure, remains constant across the section and then reduces with leakage at the other side of the bearing, to the supply pressure.

We really have only considered constantly applied loads, constant in both magnitude and direction. In many machines, however, both magnitude and direction of load are far from constant. Many machines, as in internal combustion engines, have loads which, due to a reciprocating motion, change in magnitude and direction. Examples of this type of loading are found in the connecting rod bearings, crankshaft main bearings and camshaft bearings.

When the bearing load increases or changes direction, the shaft centre will change relative to the bearing centre until all the forces are again in equilibrium. In practice, equilibrium never occurs and the shaft is continually moving its centre in relation to the bearing.

The change in radial position tends to squeeze the oil film which has already been established by hydrodynamic action. The effect of the squeezing is to try and force the oil film to flow to a new position round the shaft, but due to the viscosity of the oil, there is resistance to this change in shaft position. This resistance actually increases the load supporting capacity of the oil film - provided that the radial motion is continuous. This squeeze film component of total hydrodynamic pressure cannot last indefinitely as the load increases but it is very effective in accommodating reciprocating loads.

We have discussed briefly one type of lubrication, there are two other mechanisms which are worth of mention: hydrostatic and thin-film lubrication.

Hydrostatic Lubrication

In this type of lubrication the journal is 'floated' using oil pressure supplied by a pump. In this case there is no question of a residual oil film or the generation of a hydrodynamic oil bearing film. The pressure required is a function of the projected bearing area and the load to be carried.

Hydrostatic lubrication occurs in two basic areas:
a) for loads which are high and where the rotational speed is insufficient to establish a load bearing oil film. b) to reduce wear and to cut down starting friction, when concerned with large loads. This latter case is applicable to the starting of a turbo-generator. The heaviest rotating item is the generator rotor, which on a large machine may weigh 100 tons or ~ 1 MN. When this item is at rest there is metal to metal contact between the babbitt metal of the bearing and the steel of the shaft. High friction is present as the steel drags over the softer bearing. The jacking oil pump, supplying oil at around 14 MPa, provides sufficient force in the bearing area to lift or 'jack' the journal off the bearing. This not only reduces the initial resistance to motion but saves the bearing being unnecessarily smeared or the rotor shaft being unnecessarily abraded by the hard particles which inevitably become embedded in the softer material.

Thin Film Lubrication

Hydrodynamic film lubrication is also termed 'perfect' or 'thick' film lubrication. Thin film lubrication is obtained when some of the requirements for thick film lubrications are not wholly met. In the thick film lubrication the sliding surfaces are completely separated by a lubricant, there is no metal to metal contact and the wear is minimized. The process involved in thick film lubrication is primarily one of viscous flow; the frictional resistance being attributed to the shearing of the lubricant.

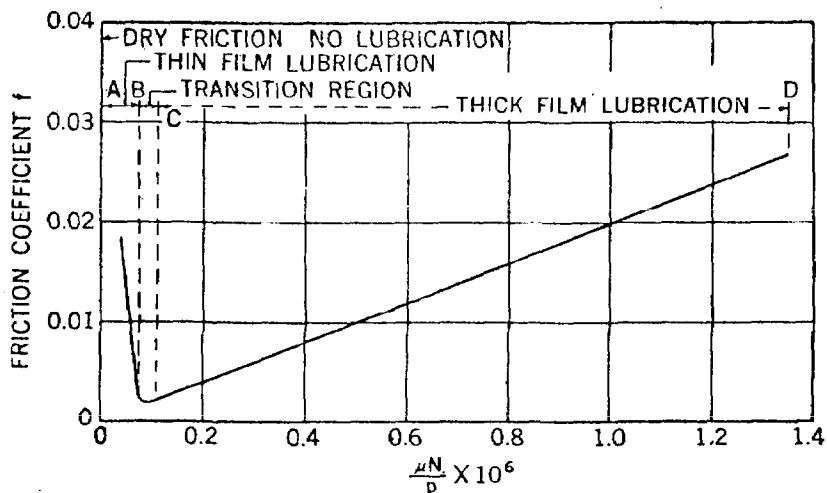
A thick film lubrication state normally only occurs at rated operating conditions. During start up the oil film may be so small that there will be metal to metal contact, wear and even local seizure. The two main reasons for this situation are that at start up the supply of lubricant is inadequate and the relative velocity is insufficient to develop the viscous pumping action necessary to establish the thick oil film.

In the thin-film or boundary region we no longer have layers of fluid with relative velocities but a single layer of fluid which may only be 2.54×10^{-9} m in thickness, ie, a molecular layer. This layer may be plating the bearing surface, either by adsorption or chemical reaction. When oils containing extreme pressure additives are used for thin-film lubrication, the film is formed by chemical reaction with the metal.

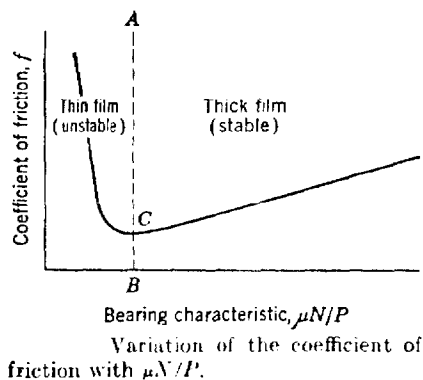
Pure mineral oil has little tendency to spread or wet but if a small amount of fatty acid, such as oleic acid, is added, the characteristics change. The combination will wet the surface more readily and adhere more strongly, the result being that there is a marked reduction in friction when the combination is used for lubrication. Obviously the process is complex but the activated lubricant molecules that are adsorbed at the bearing surface, will orient perpendicular to the surface - this increases the oiliness, which for practical purposes, is the property that causes a reduction in friction and wear in the boundary region, assuming the same viscosity and temperature of lubricating film.

The friction associated with otherwise identical bearings is a function of the load, speed and viscosity of the fluid.

Thus friction = $f \left(\frac{\mu N}{P} \right)$, this bearing characteristic may be plotted with friction to produce the curve shown.



The illustration shows how the coefficient of friction and film thickness are related to viscosity, speed and bearing load pressure. Apart from the obvious case of start up, the problems are similar on run down. It is impossible to ensure full-fluid lubrication especially when sliding speeds are low or the loads are high. Transition to boundary lubrication may be gradual. As the speed decreases or the load increases, the lubricating oil film becomes thinner and surface peaks begin to penetrate the oil film and boundary lubrication plays a more significant part with higher friction and associated heat generation. The illustration of friction against the bearing characteristic also defines the stability of the oil film.



Suppose that a bearing is operating to the right of line AB and that the coefficient of friction is caused to increase slightly. The increase in friction will create a higher temperature which causes a decrease in viscosity

which reduces the value of $\mu N/p$ and thereby reduces the friction. Thus in the thick film region we have a self correcting operation and lubrication is in a stable regime.

To the left of line AB, a decrease in viscosity would increase the friction. A temperature rise would result and the viscosity further reduced. The detrimental effects are compounded and thus thin film lubrication presents an unstable lubrication regime.

ASSIGNMENT

1. Show that when a plate is moving relative to a stationary plate and separated by a fluid the pressure distribution is $\frac{dP}{dx} = \delta S_s/\delta y$ for equilibrium conditions

Where S_s is the shear stress
 x is the horizontal distance
 y is the vertical distance

2. Upon what factors does the value of pressure in a hydrodynamic oil film depend?
3. Sketch the typical pressure distribution for a slipper bearing and explain why the maximum pressure does not occur at the narrowest section of oil film.
4. Sketch the pressure distribution of a journal bearing. What effect would you expect oil grooves to produce?
5. Describe the relative motion of a journal with respect to the bearing from stationary to full speed.
6. Describe the 'squeeze-film' mechanism and explain where this may be found.
7. Describe three types of lubrication.
8. Explain lubrication stability.
9. Would you expect the thick film flow to have a high Reynolds number?

J. Irwin Childs