Fluid Mechanics - Course 123

PARALLEL PIPE FLOW

A combination of two or more pipes connected as in Figure 6.1 so that the flow is divided among the pipes and then rejoins, is a parallel pipe system.

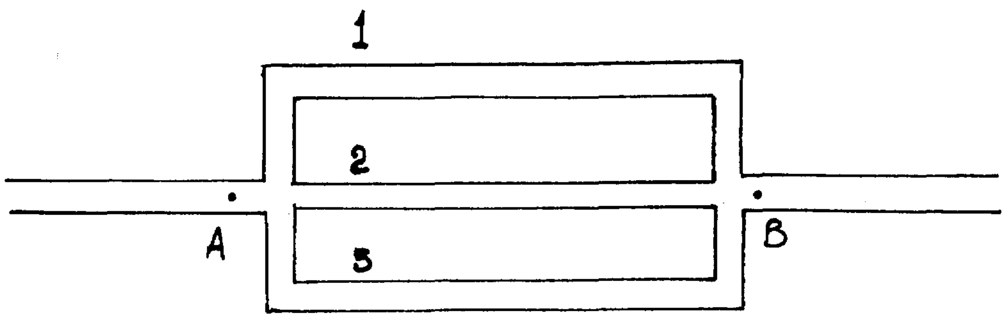

Figure 6.1

In a series pipe system the same fluid flows through all the pipes and the head losses are cumulative. In parallel pipes the losses are the same in any of the lines and the discharges are cumulative. In these situations the basic electrical analogy using Kirchoff's Laws is applicable.

In analysing parallel pipe systems, if no further information is given, it is assumed that the minor losses are added into the lengths of each pipe as equivalent lengths. Thus, in Figure 6.1, the ground condition is that:

$$E_{LOSS(1)} = E_{LOSS(2)} = E_{LOSS(3)} = \frac{P_A - P_B}{\rho} \quad J/kg$$

Example

Oil flows through the network shown in Figure 6.2 at a total rate of 1.2 m^3/s. Pressure at point A is 500 kPa(g), 'd' = 0.78, $\mu = 5 \times 10^{-4} \frac{Ns}{m^2}$

Determine the flow in each leg and the pressure at point B.

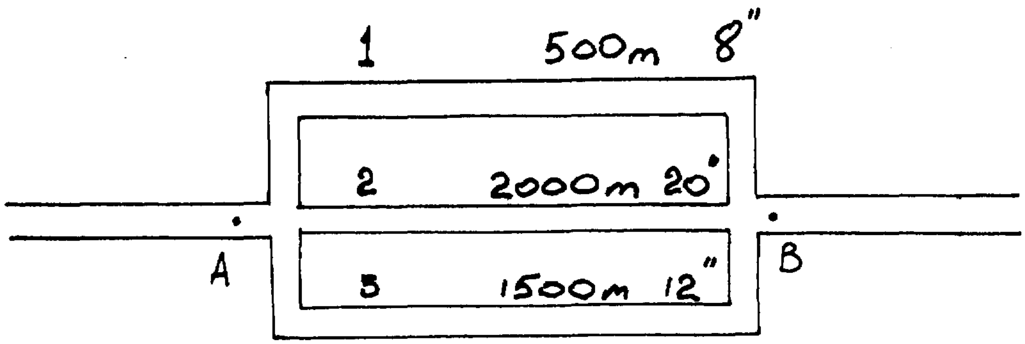

Figure 6.2

This type of question is solved by a proportionality approach and to this end a flow in one of the legs has to be assumed.

Assume flow in leg 1 is 0.2 m^3/s = Q_1^1

$$\text{Thus} \quad V_1^1 = \frac{Q_1^1}{A} = \frac{0.2 \times 10^4}{322.8} = \underline{\underline{6.2 \text{ m/s}}}$$

$$R_1^1 = \frac{VD\rho}{\mu} = \frac{6.2 \times 8 \times 2.54 \times 10^{-2} \times 0.78 \times 1000 \times 10^4}{5}$$

$$= \underline{1.97 \times 10^6}$$

$(^{\varepsilon}/D)_1$ = 0.00022 (From Figure 3.1)

f_1^1 = 0.0145 (From Figure 3.2)

Substituting $$E^1_{LOSS(1)} = \frac{f_1^1 L_1 (V_1^1)^2}{2D_1}$$

$$= \frac{0.0145 \times 500 \times 6.2^2 \times 10^2}{2 \times 8 \times 2.54}$$

$$= \underline{\underline{686 \text{ J/kg}}}$$

This loss is common to all legs and allows us to determine the flowrate in the legs.

$$E^1_{LOSS(2)} = \frac{f^1_2 L_2 (V^1_2)^2}{2D_2}$$

Thus $$V^1_2 = \left(\frac{E^1_{LOSS(2)} \times 2 \times D_2}{f^1_2 \; L_2}\right)^{\frac{1}{2}}$$

We know neither V^1_2 nor f^1_2 and an approximation has to be made to enable a flowrate to be established. From Figure 3.1 a rough value of f^1_2 may be found.

$\left(\varepsilon/D\right)_2 = 0.000095$ $\quad f^1_2$ estimated from Figure 3.1 $= 0.012$

Thus $$V^1_2 = \left(\frac{686 \times 2 \times 18.8 \times 2.54 \times 10^{-2}}{0.012 \times 2000}\right)^{\frac{1}{2}} = \underline{\underline{5.2 \text{ m/s}}}$$

Now we must check to see if the friction factor is correct.

$$R^1_2 = \frac{5.2 \times 18.8 \times 2.54 \times 10^{-2} \times 780 \times 10^4}{5}$$

$$= \underline{\underline{3.9 \times 10^6}}$$

$f^{1\,1}_2 = 0.0121$ which is well within limits

Thus $$Q^1_2 = A_2 \times V^1_2 = 1794 \times 10^{-4} \times 5.2$$

$$= \underline{\underline{0.93 \text{ m}^3/\text{s}}}$$

Similarly for pipe 3

$$E^1_{LOSS(3)} = f^1_3 \frac{L_3 \left(V^1_3\right)^2}{2D_3}$$

$$\therefore V^1_3 = \left(\frac{E^1_{Loss(3)} \times 2 \times D_3}{f^1_3 \; L_3}\right)^{\frac{1}{2}}$$

from Figure 3.1 $\left(\varepsilon/_{D}\right)_3 = 0.00015$ & $f_3^1 = 0.013$

$$\text{Thus} \quad V_3^1 = \left(\frac{686 \times 2 \times 12 \times 2.54 \times 10^{-2}}{0.013 \times 1500}\right)^{\frac{1}{2}}$$

$$= \underline{\underline{4.6 \text{ m/s}}}$$

Check value of f_3^1 $\quad R_3^1 = \frac{4.6 \times 12 \times 2.54 \times 10^{-2} \times 780 \times 10^4}{5}$

$$= \underline{\underline{2.2 \times 10^6}}$$

$f_3^{1\,1} = 0.000133$ - near enough

$$\text{Thus} \quad Q_3^1 = A_3 \times V_3^1 = 722.1 \times 10^{-4} \times 4.6$$

$$= \underline{\underline{0.33 \text{ m}^3/\text{s}}}$$

Thus $\Sigma\, Q^1$ for the assumed conditions

$$= Q_1^1 + Q_2^1 + Q_3^1 = 0.20 + 0.93 + 0.33$$

$$= \underline{\underline{1.46 \text{ m}^3/\text{s}}}$$

Hence Q_1, Q_2 and Q_3 by proportion

$$Q_1 = \frac{0.2 \times 1.2}{1.46} = \underline{0.164 \text{ m}^3/\text{s}}$$

$$Q_2 = \frac{0.93 \times 1.2}{1.46} = \underline{0.764 \text{ m}^3/\text{s}}$$

$$Q_3 = \frac{0.33 \times 1.2}{1.46} = \underline{0.271 \text{ m}^3/\text{s}}$$

We can now check the actual values of E_{LOSS}

$$V_1 = \frac{0.164 \times 10^4}{322.8} = 5.1 \text{ m/s}$$

$$R_1 = \frac{5.1 \times 8 \times 2.54 \times 10^{-2} \times 780 \times 10^4}{5}$$

$$= 1.6 \times 10^6$$

$$V_2 = \frac{0.764 \times 10^4}{1794} = 4.3 \text{ m/s}$$

$$R_2 = \frac{4.3 \times 18.8 \times 2.54 \times 10^{-2} \times 780 \times 10^4}{5}$$

$$= 3.2 \times 10^6$$

$$V_3 = \frac{0.271 \times 10^4}{722.1} = 3.8 \text{ m/s}$$

$$R_3 = \frac{3.8 \times 12 \times 2.54 \times 10^{-2} \times 780 \times 10^4}{5}$$

$$= 1.8 \times 10^6$$

$$\left(\varepsilon/_D\right)_1 = 0.00022, \quad f_1 = 0.0145$$

$$E_{LOSS(1)} = \frac{0.0145 \times 500 \times 5.1^2}{2 \times 8 \times 2.54 \times 10^{-2}}$$

$$= 464 \text{ J/kg}$$

$$\left(\varepsilon/_D\right)_2 = 0.000093, \quad f_2 = 0.0122$$

$$E_{LOSS(2)} = \frac{0.0122 \times 2000 \times 4.3^2}{2 \times 18.8 \times 2.54 \times 10^{-2}}$$

$$= 472 \text{ J/kg}$$

$$\left(\varepsilon/_D\right)_3 = 0.00015, \quad f_3 = 0.0136$$

$$E_{LOSS(3)} = \frac{0.0136 \times 1500 \times 3.8^2}{2 \times 12 \times 2.54 \times 10^{-2}}$$

$$= 483 \text{ J/kg}$$

$$\text{Average } E_{LOSS} = 473 \text{ J/kg}$$

$$\Delta P = \rho \times E_{LOSS} = 780 \times 473 = \underline{368.9 \text{ kPa}}$$

$$\text{Thus} \quad P_B = 500 - 368.9 = 131.1 \text{ kPa(g)}$$

Split Flow

In the previous example the flow through each section was determined by the pressure difference. Exactly the same situation applies when the flow is split and reaches different

Applying Bernoulli's Equation between the supply tank and the pump suction

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gh_1 + E_{ADD} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gh_2 + E_{LOSS}$$

$$\frac{101\ 300}{998.2} + 0 + 9.8 \times 6 + 0 = \frac{P_2}{998.2} + \frac{2.63^2}{2} + 0 + 4.9$$

$$\text{Thus} \quad P_2 = 101\ 300 + [9.8 \times 6 - \frac{2.63^2}{2} - 4.9]\ 998.2$$

$$= 101\ 300 + [58.8 - 3.5 - 4.9]\ 998.2$$

$$= 101\ 300 + 50\ 309$$

$$= \underline{\underline{151.609}} \text{ kPa(a)}$$

Applying Bernoulli's Equation across the pump

$$V_3 = \frac{Q_V}{A} = \frac{0.3}{872.8 \times 10^{-4}} = \underline{\underline{3.4 \text{ m/s}}}$$

$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + gh_2 + E_{ADD} = \frac{P_3}{\rho} + \frac{V_3^2}{2} + gh_3 + E_{LOSS}$$

No change in level and assume E_{LOSS} through pump is accounted for in E_{ADD}.

$$\text{Thus} \quad \frac{P_2}{\rho} + \frac{V_2^2}{2} + E_{ADD} = \frac{P_3}{\rho} + \frac{V_3^2}{2}$$

$$\text{Thus} \quad P_3 = P_2 + \rho[\frac{V_2^2}{2} - \frac{V_3^2}{2} + E_{ADD}]$$

$$= 151\ 609 + 998.2\ [\frac{2.63^2 - 3.4^2}{2} + 600]$$

$$= 151\ 609 + 998.2 \times 597.7$$

$$= \underline{\underline{748.212 \text{ kPa(a)}}}$$

Consider the discharge pipework up to the split

$$R_E = \frac{VD\rho}{\mu} = \frac{3.4 \times 13.1 \times 2.54 \times 10^{-2} \times 998.2}{1.005 \times 10^{-3}}$$

$$= \underline{\underline{1.1 \times 10^6}}$$

$$\varepsilon/_D = 0.00014 \qquad 'f' = 0.0138$$

$$E_{LOSS} = \frac{fLV_3^2}{2D} = \frac{0.0139 \times 70 \times 3.4^2}{2 \times 13.1 \times 2.54 \times 10^{-2}}$$

$$= \underline{\underline{16.9 \text{ J/kg}}}$$

Applying Bernoulli's Equation between the pump discharge and the split

$$P_3/_\rho + V_3^2/_2 + gh_3 + E_{ADD} = P_4/_\rho + V_4^2/_2 + gh_4 + E_{LOSS}$$

$$\text{Thus} \qquad P_4 = P_3 - \rho(E_{LOSS})$$

$$= 748.212 - 998.2 \times 16.9$$

$$= \underline{731.342 \text{ kPa(a)}}$$

Beyond the split both branches are open to atmosphere and the flow in each line is found from trial values using the pressure difference across the branch. The trial values recognize that the kinetic energy term is relatively insignificant when compared with the friction loss term and it is therefore ignored.

<u>Considering Branch A</u>

$$E_{LOSS} = \frac{P_4 - P_5}{\rho} - gh_5$$

$$= \frac{731\ 342 - 101\ 300}{998.2} - 9.81 \times 13$$

$$= 631.2 - 127.5$$

$$E_{LOSS} = 503.7 \text{ J/kg}$$

Thus $$\frac{fLV_A^2}{2D} = 503.7$$

We neither know 'f' nor V_A so an estimate has to be made for 'f' to allow a solution. If we consult Figure 3.1 an approximate value of 'f' is 0.014

Thus $$V_A = \left[\frac{503.7 \times 2 \times 8 \times 2.54 \times 10^{-2}}{0.014 \times 100}\right]^{\frac{1}{2}}$$

$$= \underline{\underline{12.1 \text{ m/s}}}$$

$$R_E = \frac{VD\rho}{\mu} = \frac{12.1 \times 8 \times 2.54 \times 10^{-2} \times 998.2}{1.005 \times 10^{-3}}$$

$$= \underline{2.4 \times 10^6}$$

$$\varepsilon/_D = 0.00022 \qquad f = 0.0143$$

Revised value of V_A is 12 m/s.

$$Q_A^1 = V\,A = 12 \times 322.8 \times 10^{-4}$$

$$= \underline{\underline{0.387 \text{ m}^3/\text{s}}}$$

<u>Similarly for Branch B</u>

$$E_{LOSS} = \frac{P_4 - P_5}{\rho} = 631.2 \text{ J/kg}$$

$$V_B = \left[\frac{631.2 \times 2 \times D}{f \times L}\right]^{\frac{1}{2}}$$

from Figure 3.1 $\quad$ f is 0.0135

$$\text{Thus} \quad V_B = \left[\frac{631.2 \times 2 \times 10 \times 2.54 \times 10^{-2}}{0.0135 \times 140}\right]^{\frac{1}{2}}$$

$$= \underline{13.0 \text{ m/s}}$$

$$R_E = \frac{VD\rho}{\mu} = \frac{13.0 \times 10 \times 2.54 \times 10^{-2} \times 998.2}{1.005 \times 10^{-3}}$$

$$= \underline{\underline{3.3 \times 10^6}}$$

$$\varepsilon/_D = 0.000175 \qquad f = 0.0137$$

Revised value of V_B = 12.9 m/s

$$Q_B^1 = V\,A = 12.9 \times 508.7 \times 10^{-4}$$

$$= \underline{\underline{0.656 \text{ m}^3/\text{s}}}$$

The actual values may be determined by proportion

$$Q_A = \frac{Q_A^1}{Q_A^1 + Q_B^1} \times \left(Q_A + Q_B\right)$$

$$= \frac{0.387}{0.387 + 0.656} \times (0.3) = \underline{0.111 \text{ m}^3/\text{s}}$$

Similarly $Q_B = \underline{\underline{0.189 \text{ m}^3/\text{s}}}$

ASSIGNMENT

1. Water at 15°C flows through a triple branch parallel network. The pressure at the downstream intersection is 350 kPa(g).

Line (a) is 16" SCH 40 and is 4500 m long
(b) is 14" SCH 40 and is 3000 m long
(c) is 18" SCH 40 and is 6800 m long

The flowrate is 6×10^6 kg/h.

Calculate the flow in each branch and the upstream intersection pressure. If the downstream section was open to atmosphere, what is the maximum flowrate that the network could achieve?

2. A pump delivers oil to a manifold, which is 10 m above the pump, via a 16" SCH 40 line which is 2500 m long. The line is fitted with a swing check valve, two medium radiused elbows and a fully open gate valve. The manifold feeds two tanks, one which is 600 m away and is fed with a 12" SCH 40 line; the tank has an oil level of 48 feet. The second tank is only 80 m from the manifold and is empty. Determine the pipe size to the empty tank. 20% of the flow has to enter the distant tank when the flowrate is 0.5 m^3/s from the pump. Determine the pump discharge pressure ($d = 0.95$, $\mu = 8 \times 10^{-1}$ N.S./m^2).

J. Irwin-Childs